

3.1 Measurements and their errors

3.1.1 Use of SI units and their prefixes

- SI units

Quantity	Unit	Symbol
Mass	kilogram	kg
Length	metre	m
Time	second	s
Current	ampere	A
Temperature	kelvin	K
Amount of substance	mole	Mol

- Prefixes

Name	Symbol	Multiplier
Tera	T	10^{12}
Giga	G	10^9
Mega	M	10^6
Kilo	k	10^3
Centi	c	10^{-2}
Milli	m	10^{-3}
Micro	μ	10^{-6}
Nano	n	10^{-9}
Pico	p	10^{-12}
Femto	f	10^{-15}

3.1.2 Limitation of physical measurements

- Definitions

Term	Definition
Precision of a measurement	Precise measurements = very little spread about the mean value. Depends only on the extend of random error
Precision of an instrument / resolution	The smallest non-zero reading that can be measured
Repeatability	If the original experimenter can redo the experiment with the same equipment and method and get the same results it is repeatable
Reproducibility	If the experiment is redone by a different person or with different techniques and equipment and the same results are found, it is reproducible
Accuracy	How close a measurement or answer is to the true value

- Types of errors

- Random errors
 - Affect precision, cause differences in measurements

- Cannot get rid of all random errors
- Reducing random errors
 - Take at least 3 repeats and calculate a mean
 - Use computers/data loggers/cameras to reduce human error and enable smaller intervals
 - Use appropriate equipment
 - Take large measurements
- Systematic errors
 - Affect accuracy
 - Occur due to the faults in apparatus / experimental method
 - Causes all results to be too high or too low by the **same amount** each time
 - Types
 - Zero error: balance not zeroed correctly (all increase / decrease by the same amount)
 - Parallax error: reading the scale at a different angle than parallel
 - Reducing systematic errors
 - Calibrate the apparatus by measuring a known value
 - Correct for background radiation for radiation experiments
 - Read the meniscus at eye level
 - Use controls in experiments
- Uncertainty of measurements
 - The bounds in which the accurate value can be expected to lie
 - Absolute uncertainty: uncertainty given as a fixed quantity e.g. $7 \pm 0.6 \text{ V}$
 - Fractional uncertainty: uncertainty as a fraction of the measurement e.g. $7 \pm \frac{3}{35} \text{ V}$
 - Percentage uncertainty: uncertainty as a percentage of the measurement e.g. $7 \pm 8.6\% \text{ V}$
 - Uncertainty can only be quoted to the **same precision** as the **measuring instrument / same number of decimal places as the data**
 - Work out uncertainty from the **number of decimal places** if not specified
- Methods to reduce uncertainties / errors
 - Measure larger quantities
 - Reduce percentage and fractional uncertainty
 - Repeat experiments / readings
 - Anomalies can be identified and discarded
 - Reduce random error
 - View ___ at right angle / parallel level
 - Reduce parallax error
 - Repeat in different venues / environments
 - Account of the effect of variations of the background environment
 - Use video technology / data loggers / light gates / etc.
 - Reduce random error
 - Reduce anomalies
- Reading
 - 1 value is found
 - Uncertainty in reading = \pm smallest division
- Measurement
 - The difference between 2 values are found
 - Uncertainty in measurement = $\pm 2 \times$ smallest division
- Uncertainty in different situations
 - Digital readings: uncertainty quoted or assumed to be \pm the last significant digit
 - Repeated data: uncertainty = $\pm \frac{\text{range}}{2}$
- Uncertainty calculations
 - Adding / subtracting data = add absolute uncertainties
 - Multiplying / dividing data = add percentage uncertainties
 - Raising to a power = multiply percentage uncertainty by power
 - Uncertainties given to the same number of sig figs as the data
- Uncertainties on graphs

- Uncertainties shown as error bars on graphs
- A line of best fit on a graph should go through all error bars (excluding anomalous points)
- Uncertainty of gradient of line of best fit
 - Draw a steepest and shallowest line of worst fit (must go through all error bars)
 - Calculate the gradient of the line of best and worst fit
 - The uncertainty is the difference between the best gradient and the worst gradient (the one with the greatest difference in magnitude from the 'best' line of best fit)
 - percentage uncertainty = $\frac{\text{maximum gradient} - \text{minimum gradient}}{2} \times 100\%$
- Uncertainty of x and y-intercept
 - percentage uncertainty = $\frac{\text{maximum y intercept} - \text{minimum y intercept}}{2} \times 100\%$

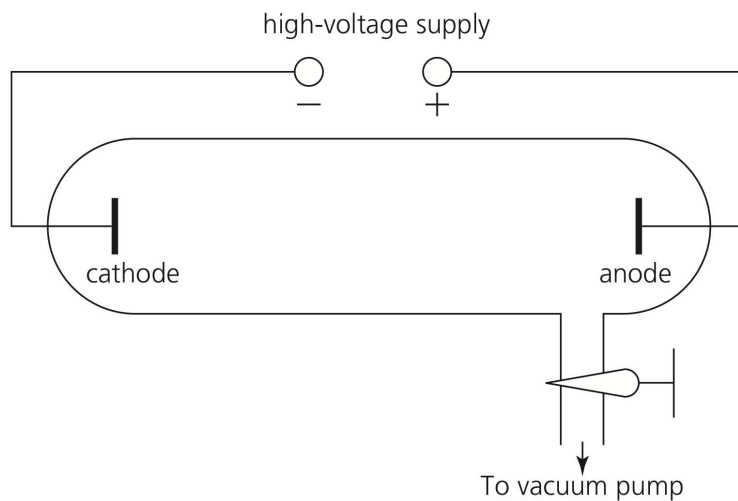
3.1.3 Estimation of physical quantities

- Orders of magnitude
 - Powers of 10 which describe the size of a value
 - Give a value to the nearest order of magnitude = round to the nearest order of magnitude

3.12.1 The discovery of the electron

3.12.1.1 Cathode rays

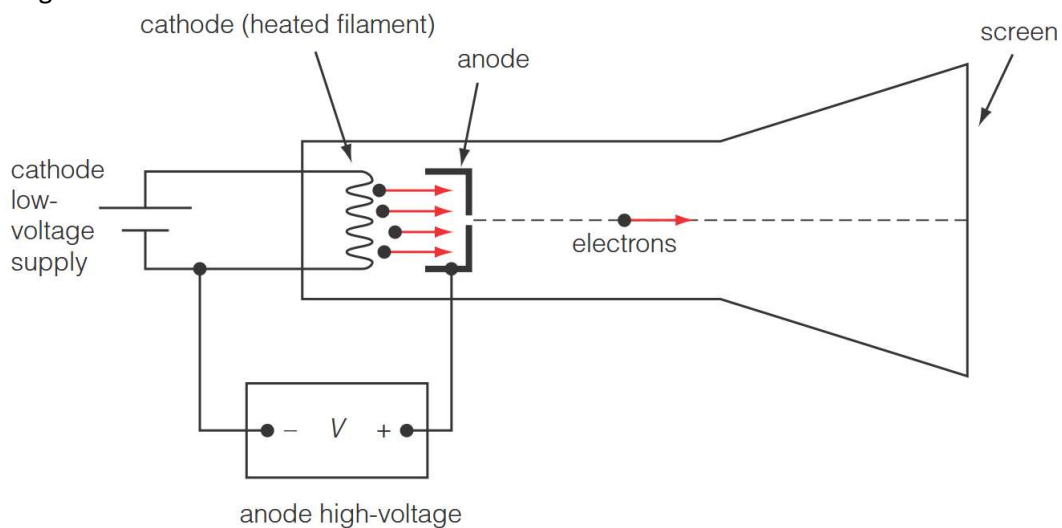
- Production of cathode rays in a discharge tube



- An electric field is produced from anode to cathode and ionise the gas atoms by pulling electrons off the gas atoms
- The gas is kept at a low pressure to create enough space to allow the positive gas ions to be accelerated through the high pd and move towards the cathode
- The gas ions strike electrons in the surface when they collide with the cathode and release more electrons
- The electrons released are accelerated along the tube from cathode to anode and collide with gas atoms causing them to become excited and de-excite releasing visible and UV photons
- If the gas pressure is further reduced the electrons ejected from the cathode travel in straight line towards the anode (known as cathode rays) and excite the atoms in the glass on striking the end of the tube
 - The body of the tube remains dark because very few gas atoms inside the tube are excited

3.12.1.2 Thermionic emission of electrons

- Electron guns

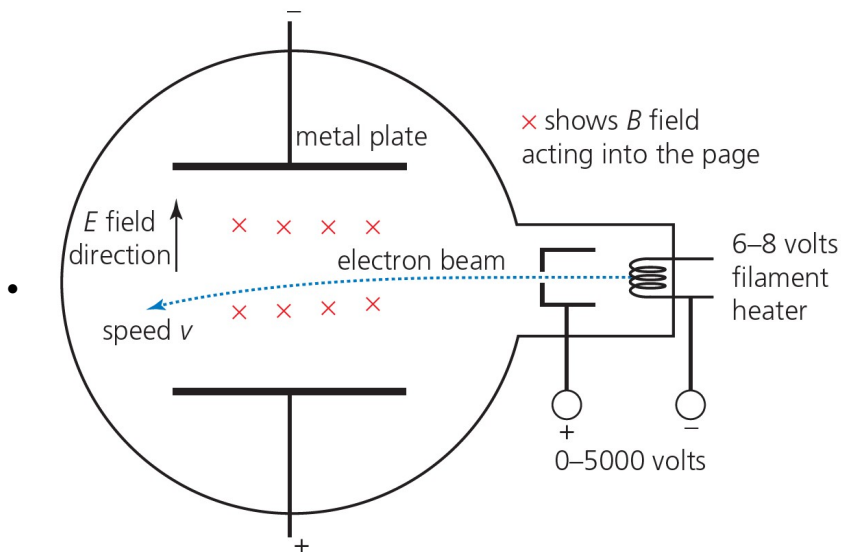


- Take place in a glass tube containing vacuum
 - There are no gas molecules for the electrons with and the tube will not glow
- Electrons are released by the cathode using thermionic emission
 - A low voltage is used (< 20V)
 - The filament is heated by the current

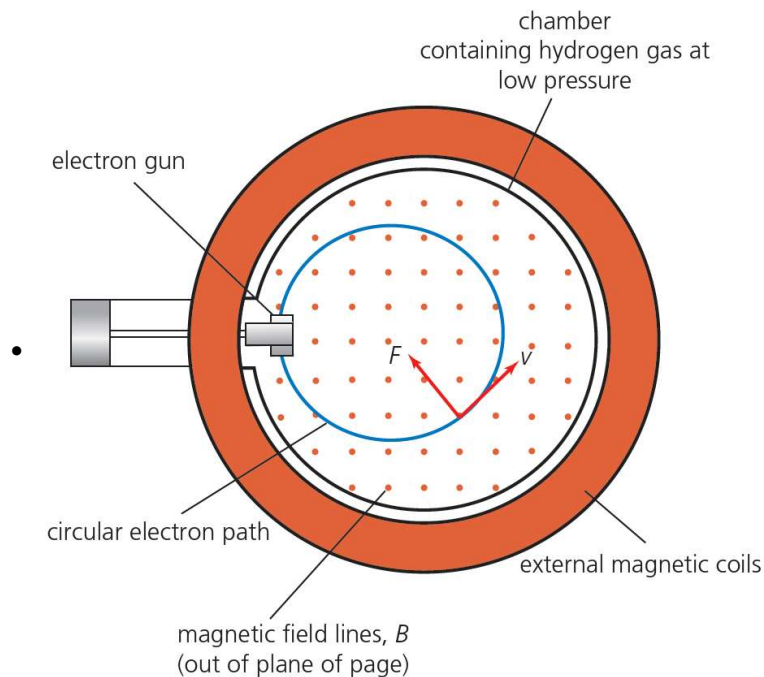
- When the metal is heated the conduction electrons in the metal start to move more vigorously and are able gain enough energy to leave the surface of the metal (energy gained > work function of the metal)
- Electrons are accelerated towards the anode using a potential difference (initial speed effectively zero)
 - An electric field from anode to cathode is formed
 - High voltage is used (> 200V)
- Electrons pass through a small hole at the anode to create a narrow beam of high speed electrons
- $E_k = \frac{1}{2}mv^2 = eV$
- Increasing current → higher filament temperature → more electrons have sufficient energy to leave the metal surface → more intense beam

3.12.1.3 Specific charge of the electron

- Thomson's crossed fields method for determining $\frac{e}{m_e}$



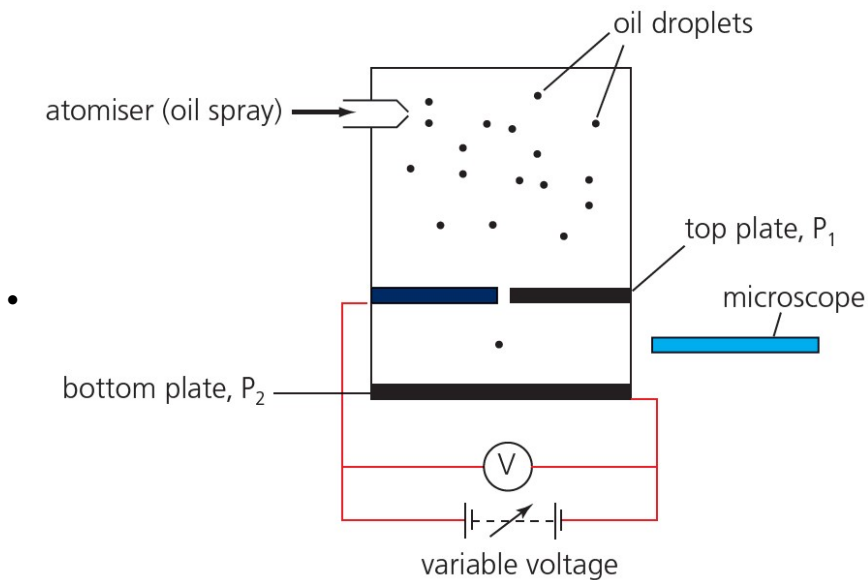
- Electrons are accelerated using an electron gun through a pd of V
- Electrons are fired into a region with two parallel plates with a pd applied through
 - There is a uniform electric field from one plate to the other
- There is a magnetic field in the region that is perpendicular to the electric field and deflects the electrons in opposite direction to the electric field
- Strength of the fields are adjusted until the electron beam passes through the crossed fields undeflected
- Force given to electron by electric field = Ee
- Force given to electron by magnetic field = Bev
- When electrons move in a straight line, the forces are balanced so $Ee = Bev$
- Therefore $v = \frac{E}{B}$
- Kinetic energy gained by electron: $\frac{1}{2}m_e v^2 = eV$
- Therefore $\frac{e}{m_e} = \frac{v^2}{2V} = \frac{E^2}{2VB^2}$
- Fine beam tube method



- Electrons are fired into a bulb with **low pressure gas** and a **uniform magnetic field passing through**
- Electrons excite the gas atoms in the bulb which de-excites releasing photons in the visible light region so its path is clearly visible
- Two large circular current-carrying coils outside and on either side of the chamber generate a uniform magnetic field at right angle to the electrons' motion
- The magnetic field provided centripetal force and electrons move in a circular path: $Bev = \frac{mv^2}{r}$
- Squaring both sides: $B^2 e^2 v^2 = \frac{m^2 v^4}{r^2}$
- Substituting $\frac{1}{2}mv^2 = eV$: $B^2 e^2 v^2 = \frac{mv^2 \times 2eV}{r^2}$
- Rearranging gives $\frac{e}{m} = \frac{2V}{r^2 B^2}$
- All the RHS quantities can be measured and used to calculate the specific charge
- Significance of Thomson's determination of $\frac{e}{m_e}$
 - Electrons have a very large specific charge
 - Specific charge of electron is about $1.76 \times 10^{11} \text{ C kg}^{-1}$
 - Specific charge of a hydrogen ion (proton) is about $9.58 \times 10^7 \text{ C kg}^{-1}$
 - The specific charge of an electron is **around 1800 times larger** than that of proton (the largest specific charge value known at the time)
 - It must have a very small mass or a very large charge
 - It showed that the **specific charge was constant** whatever gas was used to produce the electrons
 - Therefore **all atoms contain electrons**
 - Thomson went on to propose the plum pudding model

3.12.1.4 Principle of Millikan's determination of the electronic charge, e

- Stoke's Law
 - For a spherical object moving, the viscous force $F = 6\pi\eta r v$
 - η = viscosity of the fluid
 - r = radius of the object
 - v = terminal velocity of the object
- Millikan's oil drop experiment

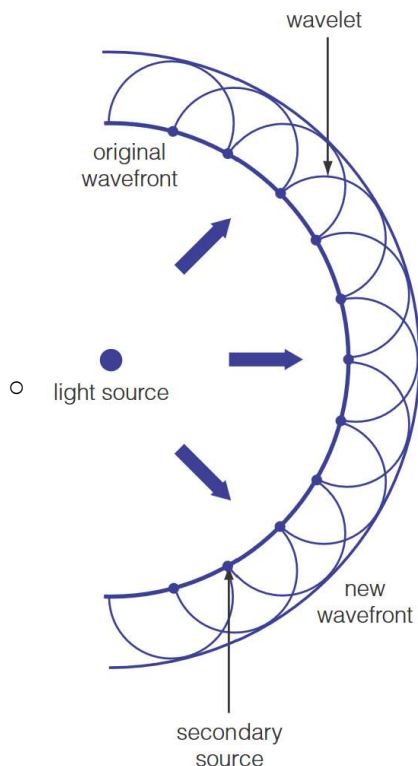


- Force and weight are balanced when the oil drops are stationary: $EQ = mg$
- Since $E = \frac{V}{d}$, $\frac{QV}{d} = mg$, so mass of the droplet is needed to determine the charge
- Using volume of sphere: $m = \frac{4}{3}\pi r^3 \rho$ so only the radius is needed
- Radius is determined by observing the motion of the same droplet when the electric field is switched off
- Only weight and resistive forces acting on the oil drop when the field is switched off → find terminal speed
- At terminal velocity v : $F = mg$
- Using Stoke's Law: $mg = 6\pi\eta r v \rightarrow \frac{4}{3}\pi r^3 \rho g = 6\pi\eta r v \rightarrow r = \sqrt{\frac{9\eta v}{2\rho g}}$
- With η and ρ known: r can be found and hence Q
- Significance of Millikan's results
 - The charge of all the oil droplets **was an integer multiple of $1.6 \times 10^{-19} \text{ C}$**
 - This is the smallest possible magnitude of charge → magnitude of charge carried by an electron

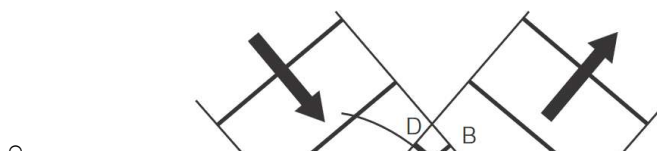
3.12.2 Wave-particle duality

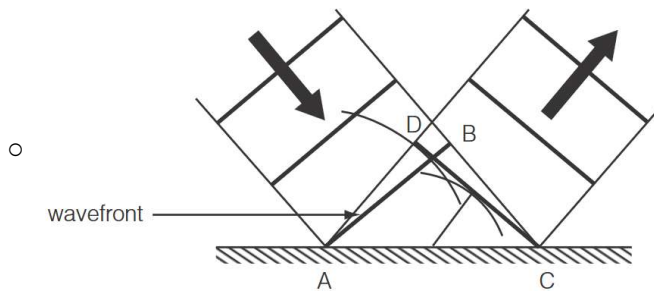
3.12.2.1 Newton's corpuscular theory of light

- Newton's corpuscular theory of light
 - Light was formed of **corpuscles (must say this)** that **travel in straight lines**
 - Reflection explanation
 - The corpuscles collide with the surface
 - A repulsive force pushes them back
 - Their component of velocity perpendicular to the surface reverses direction
 - Their component of velocity parallel to the surface stays the same
 - Refraction explanation
 - As the corpuscles approach a denser medium short-range forces of attraction causes their component of velocity perpendicular to the surface to increase
 - The parallel component of velocity stays the same
 - Light will bend towards the normal and **travel faster in denser mediums (false)**
 - **Diffraction of light cannot be explained** with Newton's theory (it was not observed at the time)
- Huygens' theory of light
 - **Light behave as (longitudinal) waves**
 - Every point on a wavefront is a point source of secondary wavelets which spread out to form the next wavefront

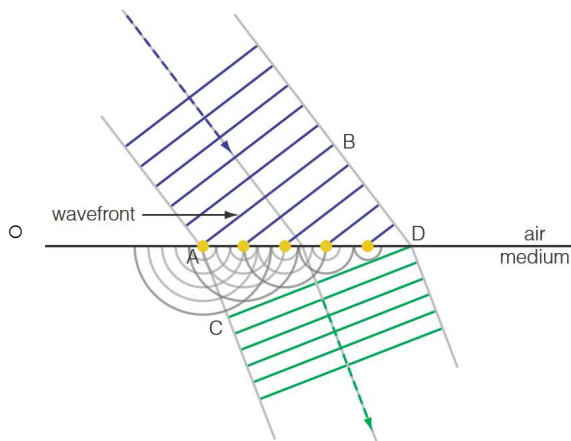


- Reflection explanation
 - The whole wavefront will not reach the surface at once unless travelling perpendicular to the surface which
 - The point at which the wavefront is reflected acts as a secondary source and new wavelets are formed
 - Wavelets spread away from the source once they react it and rejoin with others to reform the reflected wavefront





- Refraction
 - **Assumed that light travels slower in a denser medium**
 - As it entered a more optically dense medium it would slow down and therefore bends towards the normal



- Why Newton's theory was preferred
 - Newton had a much stronger scientific reputation than Huygens
 - It was not possible to measure the speed of light in air / transparent substance at the time so no experimental evidence to determine whether light travels faster in air or a transparent substance
 - Diffraction had not yet been observed so there was no evidence to disprove Newton's theory

3.12.2.2 Significance of Young's double slit experiment

- Young's double slit experiment
 - Coherent light is shown through 2 slits so that it diffracts
 - Each slit acts as a coherent light source
 - The light from the two slits superpose on the screen
 - There is a path difference for the light from each slit due to different distances between a point on the screen and the two slits
 - Light fringes formed when the path difference is an integer number of wavelengths → meet in phase and interfere constructively
 - Dark fringes formed when the path difference is a whole number and a half of wavelengths → meet in anti-phase and interfere destructively
- Significance of its result
 - According to Newton's theory the 2 slits would just produce 2 bright fringes corresponding to themselves instead of an interference pattern (particles don't diffract)
 - According to Huygen's wave theory an interference pattern would be produced
 - **Diffraction and interference are wave properties → light must be a wave**
 - Young accepted Huygen's idea that there must exist an ether to act as a medium to carry the light waves
 - Huygen's theory widely wasn't widely accepted until the speed of light was measured
 - It was discovered that light travels slower in water than air as contrary to Newton's predictions
 - The acceptance was delayed due to Newton's historical prestige so that the scientists expected Newton to be correct

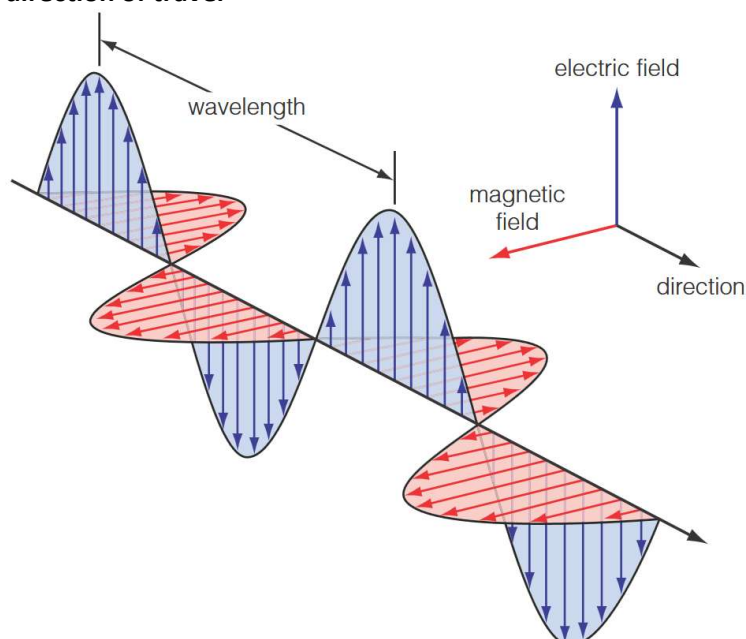
3.12.2.3 Electromagnetic waves

- Types of electromagnetic waves

Region	Wavelength range / m	Origin
Radio waves	10^{-1} to 10^6	Oscillations in electric fields
Microwaves	10^{-3} to 10^{-1}	Molecular interactions
Infrared (IR)	10^{-7} to 10^{-3}	
Visible	4×10^{-7} to 7×10^{-7} (400–700 nm)	Energy transitions within atoms
Ultraviolet (UV)	10^{-9} to 10^{-7}	
X-rays	10^{-13} to 10^{-8}	
Gamma rays	10^{-16} to 10^{-10}	Nuclear de-excitation

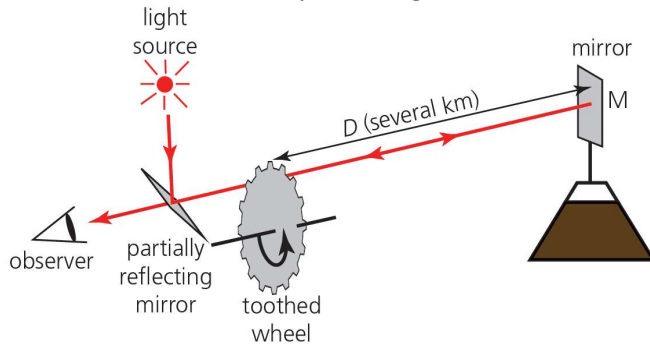
- Maxwell's theory of electromagnetic waves

- An electromagnetic wave can exist when a changing electric and magnetic field each continuously induce the other
- It can travel through space at a transverse wave with the electric and magnetic field components oscillating **in phase + perpendicular to each other + perpendicular to and to the direction of travel**



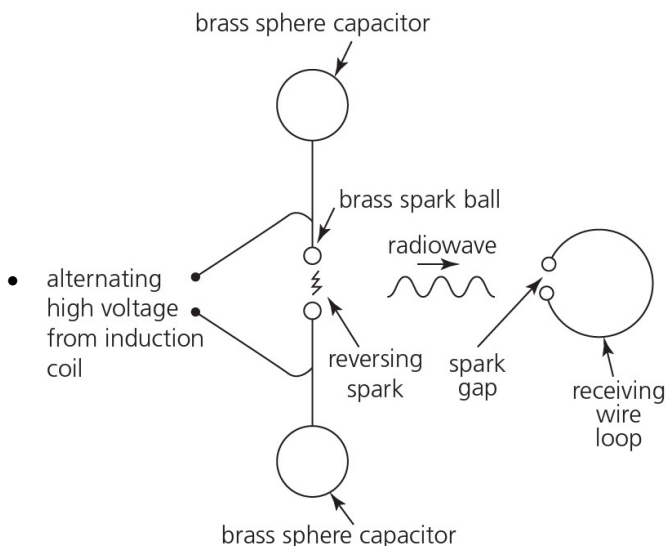
- Maxwell's formula for the speed of electromagnetic waves in vacuum

- $c = \frac{1}{\sqrt{\mu_0 \epsilon_0}}$
- μ_0 = permeability of free space
 - Relates the magnetic flux density of a magnetic field **in free space** to the electric current that creates it
 - Unit H = henry, $1 \text{ H} = 1 \text{ VsA}^{-1}$
- ϵ_0 = permittivity of free space
 - Relates the electric field strength **in free space** to the charge that creates it
 - Unit F = farad, $1 \text{ F} = 1 \text{ C V}^{-1}$
- Fizeau's determination of the speed of light



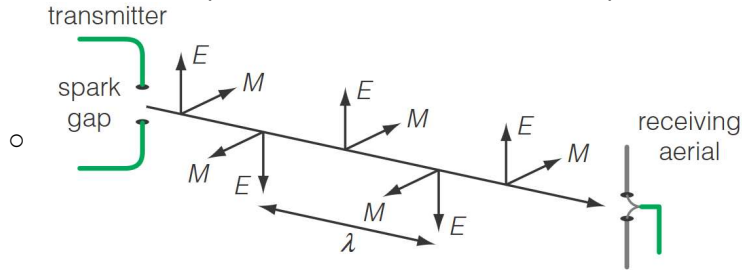
- Shine light beam at a partially reflecting mirror
- Light reflected towards the distant mirror several kilometres (distance of D) away
- Light passes between the teeth of a rotating toothed wheel with N teeth and N gaps of equal widths to the mirror
- Mirror reflects the light and light travels back through the teeth of the wheels again
- The minimum frequency f at which no reflected light can be seen is recorded
- At this frequency the time it takes to travel through the width of one tooth / gap is the same as the time it takes for the light to travel from the wheel to the mirror and back
- $t = \frac{T}{2N} = \frac{1}{2Nf} = \frac{2D}{c}$
- $\therefore c = \frac{2D}{(2Nf)^{-1}} = 4DNf$

- Hertz's discovery of radio waves



- Electromagnetic waves could be produced by a spark created by an electrical discharge
- Radio waves are produced when high voltage sparks jump across a gap of air
 - Oscillating current in the gap indicates the presence of an oscillating electric field
 - Oscillating current in the gap produces an oscillating magnetic field
- The radio waves can be detected with a loop of wire with a gap
 - This will detect the waves' alternating magnetic field as the field will enter the loop causing a change in the magnetic flux
 - Emf is induced in the wire loop \rightarrow current induced
 - Sparks are created between the brass spheres of the detector

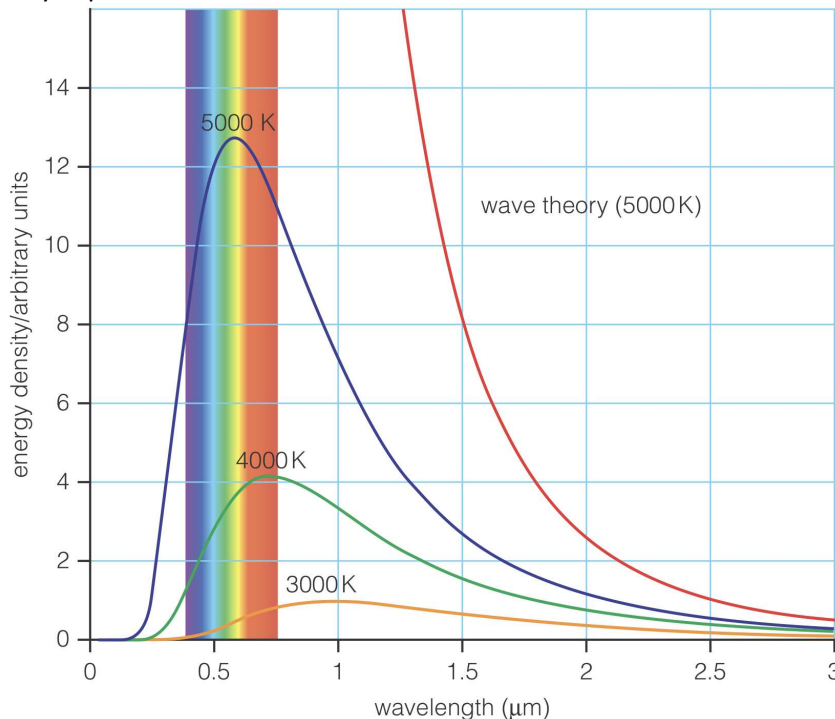
- Waves emitted are polarised as the electrons in the spark move in one plane



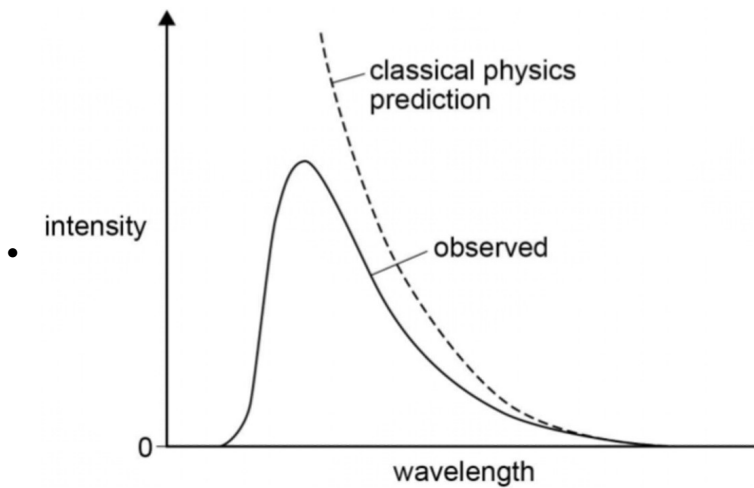
- The signal strength gradually reduces from a maximum value when the receiving gap is in line with the spark gap to zero when it is perpendicular to the gap
- Radio waves could be reflected by a metal sheet → stationary waves are created
 - Wavelength can be found by measuring the distance between adjacent nodes (half a wavelength)
 - Speed calculated by Hertz ($c = f\lambda$) was found to be the same as the one predicted by Maxwell → radio waves are EM waves
- Radio waves pass through insulators / focused by a concave reflector

3.12.2.4 The discovery of photoelectricity

- Black body
 - An object that absorbs or emits **all wavelengths of electromagnetic radiation** falling on the object
 - It does not reflect any wavelengths
 - The emission spectrum (how much energy is emitted at each wavelength) depends only on the absolute temperature (T) of the object
- Blackbody radiation
 - Electromagnetic radiation emitted by a body that has a **continuous spectrum** whose **shape depends only on the body's absolute temperature**
- Black body experimental results



- The intensity increases with increasing temperature
 - The total power of the emitted radiation (area under the curve) emitted increases with increasing temperature
 - The peak wavelength decreases with increasing temperature
- Ultraviolet catastrophe



- Prediction with wave theory
 - As the wavelength of the radiation decreases, the intensity of the radiation would increase
 - Leads to a prediction of an infinite amount of UV radiation being emitted
 - **Not supported by experimental evidence**
- Comparison to experimental evidence
 - The intensity is similar at long wavelengths
 - The wave theory predicts intensity to continue increasing at shorter wavelengths
 - The experiment showed a peak intensity and the intensity decreases with decreasing wavelength after the peak
- Could be resolved by using Planck's interpretation of electromagnetic waves
 - EM radiation is emitted in **quanta** and is quantised
 - Energy of a quantum of EM energy is directly proportional to frequency and is given by $E = hf$
- Wave theory's problem with photoelectricity
 - There is no photoemission below the threshold frequency even with bright light
 - Wave theory would allow gradual accumulation of energy to cause emission
 - Any frequency of light should be able to cause electron emission
 - Electrons are emitted with no noticeable decay
 - In wave theory time would elapse while an electron gains sufficient energy to leave the surface
 - Intensity of the light does not affect the KE of the emitted electrons
 - High intensity waves would be expected to give higher KE to an electron
- Einstein's explanation of photoelectricity with photons
 - When light is incident on a metal surface an electron at the surface absorbs a **single photon** from the incident light and gains energy equal to hf
 - All of the photon's energy is transferred to the electron
 - All electrons receive the same amount of energy
 - The energy is transferred to the electron immediately when they interact → photoelectrons emitted immediately
 - An electron can leave the metal surface if the energy gained > the work function of the metal
 - Electrons deeper in the metal need more energy as they lose energy via collisions
 - Excess energy gained becomes KE of the photoelectron
- Significance of Einstein's explanation
 - Established the wave-particle duality of light
 - Proved Planck's 'quanta' model of electromagnetic radiation

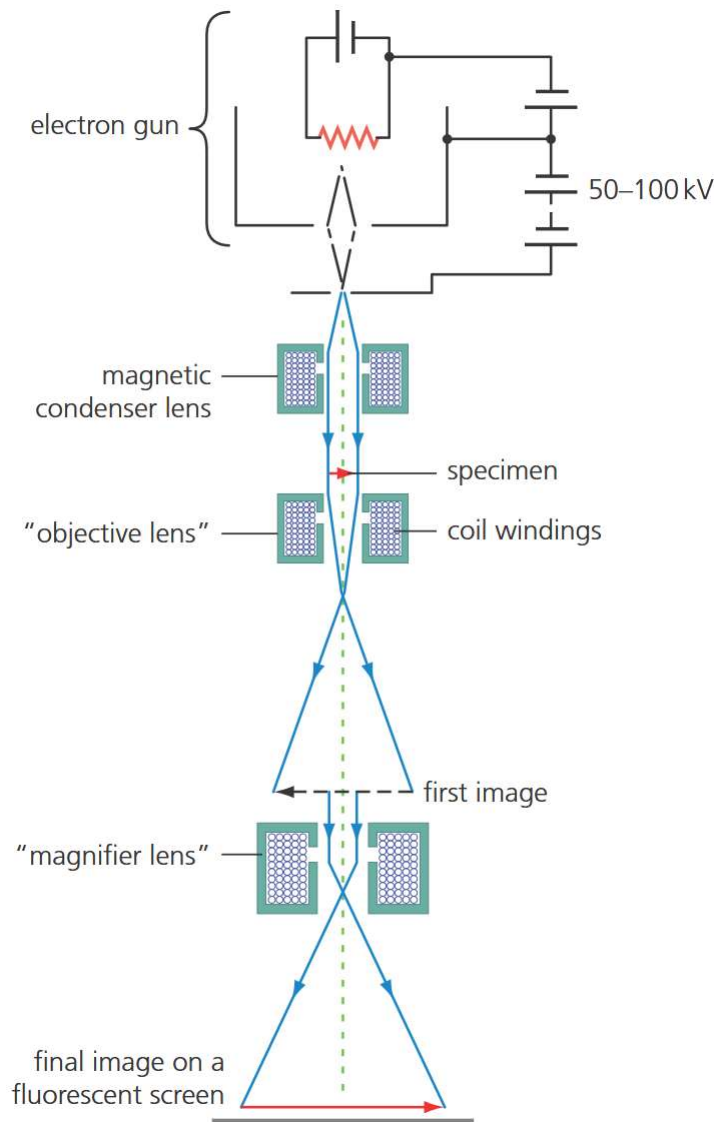
3.12.2.5 Wave-particle duality

- De Broglie wavelength
 - $\lambda = \frac{h}{p} = \frac{h}{mv}$
- Electron diffraction
 - Provided experimental evidence for de Broglie's prediction of the existence of matter waves

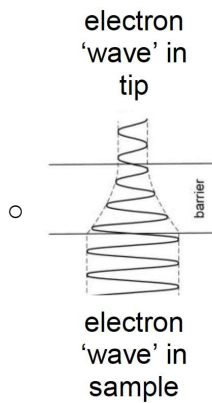
- If electrons were particles there will simply be a bright spot on the screen as electrons are diffracted randomly
- Diffraction is a wave property
- Electrons are fired towards a crystal lattice with an electron gun
- We can control the de Broglie wavelength of the electrons emitted by adjusting the pd across the anode
 - $\frac{1}{2}mv^2 = eV$
 - $m^2v^2 = 2meV$
 - $mv = \sqrt{2meV}$
 - $\lambda = \frac{h}{\sqrt{2meV}}$
 - Higher pd = shorter wavelength
- The electrons interacted with the small gaps between atoms and formed a diffraction pattern on a fluorescent screen behind the crystal
- As pd increase the de Broglie wavelength of electron decreases so the fringe spacing decreases (and vice versa)
- This follows wave theory which states that fringe spacing increases as wavelength increases

3.12.2.6 Electron microscopes

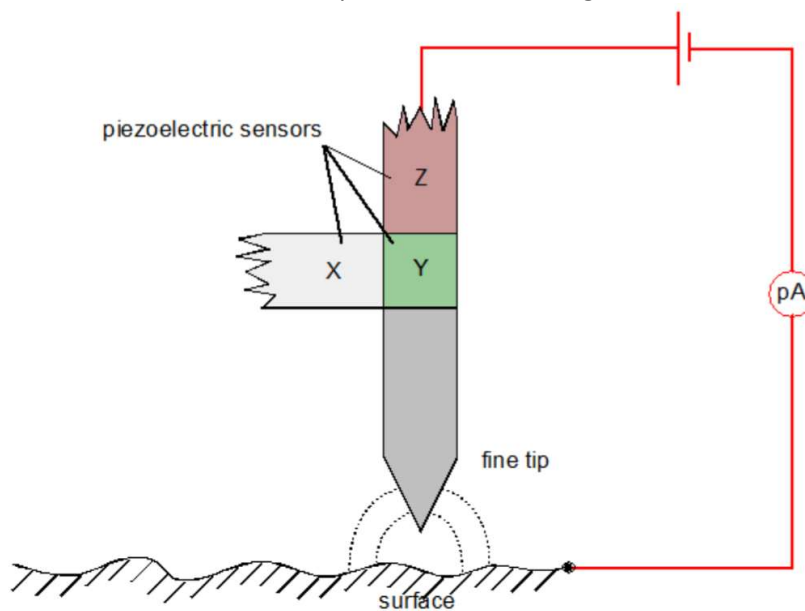
- Voltage used
 - The de Broglie wavelength of the electron should be of the order of the size of the atom to have enough resolving power
 - About 0.1 nm = 10^{-10} m
 - $\lambda = \frac{h}{\sqrt{2meV}} \rightarrow V = \frac{h^2}{2me\lambda^2}$
- Transmission electron microscope (TEM)



- The sample must be **extremely thin** so the electrons do not slow down and wavelength do not change
- Electrons pass through 3 lenses
 - Condenser lens deflects: deflects the electrons into a wide parallel beam incident uniformly on the sample
 - Objective lens: forms an image of the sample above it
 - Projector lens: magnify the image made by the objective lens and project it onto the fluorescent screen
 - Electrons through the middle of the lenses are undeviated
 - Electrons on the edges are deflected by magnetic fields toward the axis of the TEM
- Higher accelerating voltage = lower wavelength = higher resolving power
- Limited resolving power
 - Wavelength depends on speed of the electrons
 - The sample thickness reduces the speed of the electrons → increasing the wavelength and decreasing the detail
 - Electrons travel at a range of speeds since some lose KE while leaving the source metal so they have different wavelengths and different diffraction patterns causing blurred image
- Scanning tunnelling microscope (STM)
 - Uses quantum tunnelling of electrons to form an image of the object surface
 - Electrons in the sample surface face a potential barrier in the gap that they have insufficient energy to overcome classically
 - Because electrons have wave properties there is a probability that some of them cross the gap from the sample to the tip if the physical / potential gap is small enough
 - Smaller gap = increased likelihood of quantum tunnelling occurring



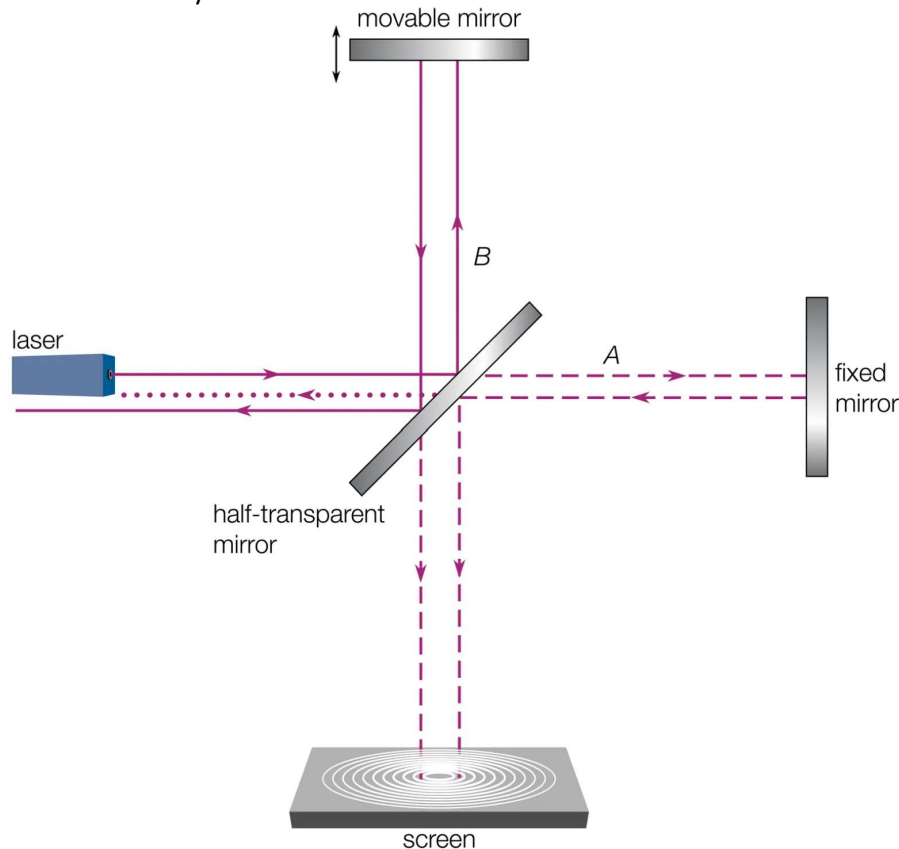
- Has a very fine tipped probe which moves across the surface and stays at a constant potential
 - Electrons can only travel in one direction
- As the probe moves across the surface the size of gap between probe and surface varies and the tunnelling current changes
 - Increases if the gap becomes narrower
 - Decreases if the gap becomes larger
- Constant height mode
 - The probe is kept at a constant height
 - The current is recorded and used to image the object surface
- Constant current mode
 - The current is kept constant
 - When probe move over higher layer of electrons current increases → tip is moved higher to reduce current to the original value
 - When probe move over lower layer of electrons current decreases → tip is moved lower to reduce current to the original value
 - The movement of the probe is used to image the surface



3.12.3 Special relativity

3.12.3.1 The Michelson-Morley experiment

- The ether
 - It was believed that light travels in medium call ether
 - The speed of light in ether is constant
 - Sun was thought of being approximately at rest reloative to the ether
 - The Earth was believed to be moving relative to the ether
 - Speed of light depends on the Earth's velocity relative to the ether and its direction to Earth's velocity
- The Michelson-Morley interferometer



- Monochromatic laser is used
- The half-transparent mirror splits the beam of monochromatic light into two beams travelling in two different directions
- One direction is set to be moving parallel to Earth's motion and the other side is perpendicular to Earth's motion
- The same distance is travelled by light deflected to both directions before reaching the screen
- The two lights combine at the screen and an interference pattern is created on the screen
- Expected result
 - The time taken for the light travel in the 2 tracks should be different to each other as their speed is different
 - Light moving parallel to Earth's motion will take longer to travel
 - Light moving perpendicular to Earth's motion has its speed unaffected
 - Expected to observe a small but observable shift of the interference fringe pattern (phase difference)
 - The absolute motion of Earth would be detected / ether will be shown to exist
- Observed result
 - No shifts in interference pattern were observed (even when rotating the apparatus)
 - Conclusion: **the speed of light was the same whether the observer was moving relative to the source or not**
 - The experiment showed that there is no absolute motion (which the ether could have

- acted as)
 - No evidence of ether → light travels without the need for a material medium

3.12.3.2 Einstein's theory of special relativity

- Inertial frame of reference
 - A reference frame in which Newton's Laws hold
 - Moves at a constant velocity (**requires no acceleration**)
- Einstein's postulates of special relativity
 - Physical laws have the same form in all inertial frames
 - The speed of light in free space is invariant and independent of the motion of the source or observer

3.12.3.3 Time dilation

- Muon decay experiment
 - Using a Geiger counter to detect cosmic muons at an altitude at 2km and at sea level
 - Half-life of muons = 1.5 μs → most should decay before reaching sea level as it takes 6.7 μs to travel 2km at muon's speed
 - Reality: 80% reached sea level
- Lorentz factor
 - $$\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$$
- Time dilation
 - $$t = \gamma t_0 = \frac{t_0}{\sqrt{1 - \frac{v^2}{c^2}}}$$
 - t is time interval measured by reference frame **moving** relative to object being observed
 - t_0 is time interval measured by reference frame **stationary** relative to object being observed (proper time)
- Muon g-2 experiment explanation with time dilation
 - The time measured by muon (stationary relative to muon) is much shorter than time measured by human (moving relative to muon)
 - Much shorter time to decay than as measured by human

3.12.3.4 Length contraction

- Length contraction
 - A ruler / pen moving in the same direction as its length **appears shorter than when it is stationary**
 - $$l = \frac{l_0}{\gamma} = l_0 \sqrt{1 - \frac{v^2}{c^2}}$$
 - l is length measured by a reference frame **moving** relative to the object
 - l_0 is length measured by a reference frame **stationary** relative to the object (proper length)
- Muon g-2 experiment explanation with length contraction
 - The length from atmosphere to ground measured by muon (moving relative to Earth) is much shorter than the distance measured by human (stationary relative to Earth)
 - Much shorter time is needed to travel the distance

3.12.3.5 Mass and energy

- Relativistic mass
 - $$m = \gamma m_0 = \frac{m_0}{\sqrt{1 - \frac{v^2}{c^2}}}$$
 - m_0 = proper mass observed in a reference frame stationary relative to the object
 - m = larger mass observed in a reference frame moving relative to the object

- Relativistic energy
 - Total energy of an object $E = mc^2 = \gamma m_0 c^2$
 - When at rest: rest energy $E_0 = m_0 c^2$
 - Transferring energy to an object increases its mass (rest energy increases if KE doesn't increase)
 - Transferring energy from an object reduces its mass
 - The difference between the 2 energies is the KE of the object
 - $E_k = E - E_0 = (\gamma - 1)m_0 c^2$
 - When $v \ll c$, $E_k \approx \frac{1}{2} m_0 v^2$
- Bertozzi's experiment
 - Used a particle accelerator to carry out an experiment to investigate the relationship between the velocity of an electron (v) and its kinetic energy (E_k)
 - Plotted a graph of $\frac{v^2}{c^2}$ against kinetic energy
 - Results aligned with Einstein's theoretical predictions from special relativity
 - As KE increases the electron's speed approaches a limiting value equal to the speed of light

