

3.1 Measurements and their errors

3.1.1 Use of SI units and their prefixes

- SI units

Quantity	Unit	Symbol
Mass	kilogram	kg
Length	metre	m
Time	second	s
Current	ampere	A
Temperature	kelvin	K
Amount of substance	mole	mol

- Prefixes

Name	Symbol	Multiplier
Tera	T	10^{12}
Giga	G	10^9
Mega	M	10^6
Kilo	k	10^3
Centi	c	10^{-2}
Milli	m	10^{-3}
Micro	μ	10^{-6}
Nano	n	10^{-9}
Pico	p	10^{-12}
Femto	f	10^{-15}

3.1.2 Limitation of physical measurements

- Definitions

Term	Definition
Precision of a measurement	Precise measurements = very little spread about the mean value. Depends only on the extend of random error
Precision of an instrument / resolution	The smallest non-zero reading that can be measured
Repeatability	If the original experimenter can redo the experiment with the same equipment and method and get the same results it is repeatable
Reproducibility	If the experiment is redone by a different person or with different techniques and equipment and the same results are found, it is reproducible
Accuracy	How close a measurement or answer is to the true value

- Types of errors

- Random errors
 - Affect precision, cause differences in measurements

- Cannot get rid of all random errors
- Reducing random errors
 - Take at least 3 repeats and calculate a mean
 - Use computers/data loggers/cameras to reduce human error and enable smaller intervals
 - Use appropriate equipment
 - Take large measurements
- Systematic errors
 - Affect accuracy
 - Occur due to the faults in apparatus / experimental method
 - Causes all results to be too high or too low by the **same amount** each time
 - Types
 - Zero error: balance not zeroed correctly (all increase / decrease by the same amount)
 - Parallax error: reading the scale at a different angle than parallel
 - Reducing systematic errors
 - Calibrate the apparatus by measuring a known value
 - Correct for background radiation for radiation experiments
 - Read the meniscus at eye level
 - Use controls in experiments
- Uncertainty of measurements
 - The bounds in which the accurate value can be expected to lie
 - Absolute uncertainty: uncertainty given as a fixed quantity e.g. $7 \pm 0.6 \text{ V}$
 - Fractional uncertainty: uncertainty as a fraction of the measurement e.g. $7 \pm \frac{3}{35} \text{ V}$
 - Percentage uncertainty: uncertainty as a percentage of the measurement e.g. $7 \pm 8.6\% \text{ V}$
 - To reduce percentage and fractional uncertainty: measure larger quantities
 - ★ • Uncertainty can only be quoted to the **same precision** as the **measuring instrument / same number of decimal places as the data**
 - Work out uncertainty from the **number of decimal places** if not specified
- Reading
 - 1 value is found
 - Uncertainty in reading = \pm smallest division
- Measurement
 - The difference between 2 values are found
 - Uncertainty in measurement = $\pm 2 \times$ smallest division
- Uncertainty in different situations
 - Digital readings: uncertainty quoted or assumed to be \pm the last significant digit
 - Repeated data: uncertainty = $\pm \frac{\text{range}}{2}$
- Uncertainty calculations
 - Adding / subtracting data = add absolute uncertainties
 - Multiplying / dividing data = add percentage uncertainties
 - Raising to a power = multiply percentage uncertainty by power
 - Uncertainties given to the same number of sig figs as the data
- Uncertainties on graphs
 - Uncertainties shown as error bars on graphs
 - A line of best fit on a graph should go through all error bars (excluding anomalous points)
- Uncertainty of gradient of line of best fit
 - Draw a steepest and shallowest line of worst fit (must go through all error bars)
 - Calculate the gradient of the line of best and worst fit
 - The uncertainty is the difference between the best gradient and the worst gradient (the one with the greatest difference in magnitude from the 'best' line of best fit)
 - percentage uncertainty = $\frac{\text{maximum gradient} - \text{minimum gradient}}{2} \times 100\%$
- Uncertainty of x and y-intercept
 - percentage uncertainty = $\frac{\text{maximum y intercept} - \text{minimum y intercept}}{2} \times 100\%$

3.1.3 Estimation of physical quantities

- Orders of magnitude
 - Powers of 10 which describe the size of a value
 - Give a value to the nearest order of magnitude = round to the nearest order of magnitude

3.2.1 Particles

3.2.1.1 Constituents of the atom

- Constituents of an atom

Particle	Charge (C)	Relative charge	Mass (kg)	Relative Mass	Specific Charge (Ckg ⁻¹)
Proton	$+1.6 \times 10^{-19}$	+1	$1.67(3) \times 10^{-27}$	1	9.58×10^7
Neutron	0	0	$1.67(5) \times 10^{-27}$	1	0
Electron	-1.6×10^{-19}	-1	9.11×10^{-31}	0.0005	1.76×10^{11}

- Specific charge

- specific charge = $\frac{\text{charge}}{\text{mass}}$

- Unit = C kg⁻¹

- Nuclide notation

- A_ZX

- A = nucleon / mass number = number of nucleons (proton + neutron)

- Z = proton / atomic number = number of protons

- X = symbol for the element

- Isotopes

- Atoms with the same number of **protons and electrons** but **different numbers of neutrons & different atomic masses**

- Nuclide

- A type of nucleus

- Atom and nucleus size

- Size of atom $\approx 10^{-10}$ m

- Size of nucleus $\approx 10^{-15}$ m = 1 fm

3.2.1.2 Stable and unstable nuclei

- The strong nuclear force (SNF)

- Only affects hadrons

- Acts on nucleons

- Keeps the nucleus stable by counteracting the electrostatic force of repulsion between protons in the nucleus and keeping protons and neutrons together

- Prevents proton and neutron from moving closer or further apart

- Very short range

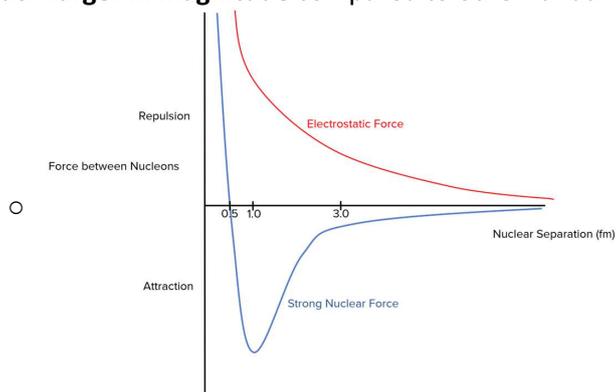
- **Repulsion** for separations less than about **0.5 fm**

- **Attraction** for separations up to **3 fm** (1 fm = 10^{-15} m)

- Negligible beyond **3 fm**

- Exchange particles = pions / gluons

- Much larger in magnitude** compared to other fundamental forces



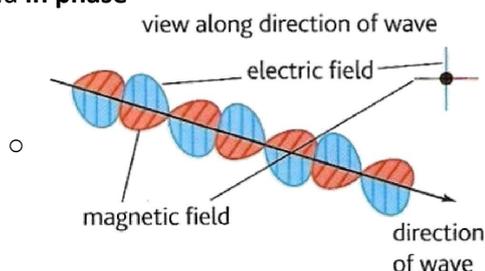
- Unstable nuclei

- Too many protons and / or neutrons

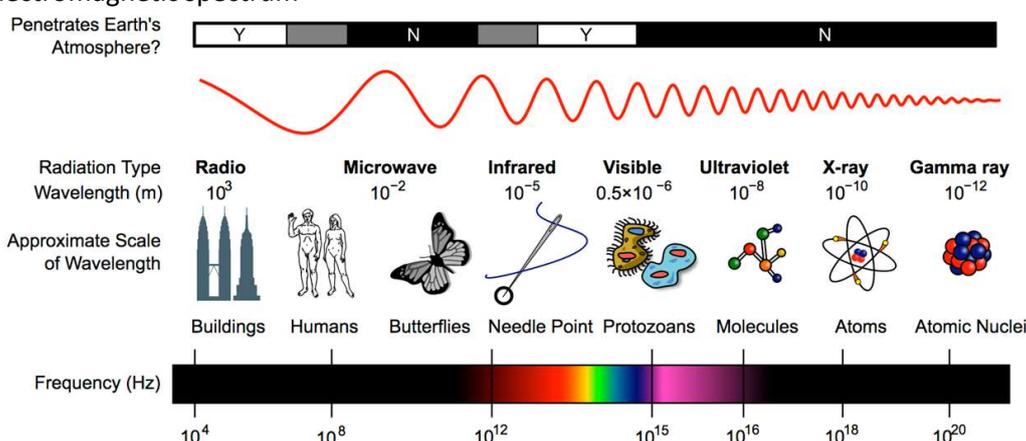
- SNF not enough to keep them stable
- Decay in order to become stable (type depends on the amount of each nucleon)
- Alpha decay
 - **Too many protons and neutrons**
 - Alpha particle emitted (2 protons + 2 neutrons)
 - ${}^A_ZX \rightarrow {}^{A-4}_{Z-2}Y + {}^4_2\alpha$
- Beta-minus decay
 - **Too many neutrons** (neutron-rich)
 - A neutron changes into a proton
 - Fast-moving electron (beta particle) + an electron antineutrino (antiparticle with no charge) emitted
 - ${}^A_ZX \rightarrow {}^{A}_{Z+1}Y + {}^0_{-1}\beta + \bar{\nu}_e$
- Neutrino (ν)
 - At first scientists believed that only an electron was emitted from the nucleus during beta-minus decay
 - Observation of energy levels before + after decay showed that energy was not conserved (some energy was lost)
 - Neutrinos were hypothesised for the loss of energy and later observed
- Why is collaboration important to making advances in understanding of particle physics
 - Results of experiments must be independently checked/validated/peer reviewed before they are accepted/can be confirmed
 - Particle accelerators are very expensive and collaboration helps to spread the cost of building them
 - Many skills and disciplines are required which one team are unlikely to have
 - Lots of data to process so more teams are needed

3.2.1.3 Particles, antiparticles and photons

- EM radiation
 - Emitted when **charged particles lose energy**
 - When a fast-moving electron is stopped / slows down / changes direction
 - When an electron in a shell of an atom moves to a different shell of lower energy
 - Consists of two linked electric and magnetic field waves that are at **right angles** to each other and **in phase**



- The electromagnetic spectrum



- Photon model of EM radiation
 - EM waves are emitted as short burst of waves in different directions

- Each burst = a packet of EM waves = a photon
- Photons transfer energy and have **no mass**
- The energy of the photons is directly proportional to the frequency of EM radiation
 - $E = hf = \frac{hc}{\lambda}$
 - h = Planck constant = 6.63×10^{-34} Js

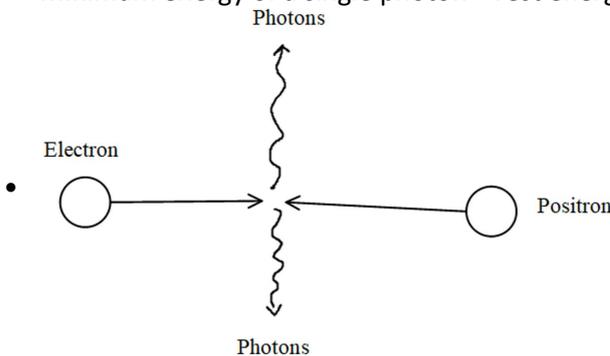
- Electron volts
 - 1 electron volt = the energy transferred when an electron is moved through a p.d. of 1 V
 - 1 eV = 1.60×10^{-19} J (1 MeV = 1.60×10^{-13} J)

- Antiparticle
 - All particles of normal matter have a corresponding antiparticle
 - **Same rest energy and mass** as the particle
 - **All other properties are opposite** e.g. charge / strangeness
 - Will **undergo annihilation** with the normal particle if they meet

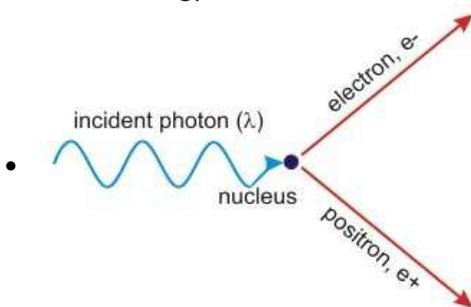
- Types of antiparticles

Particle	Antiparticle
Electron	Positron
Proton	Antiproton
Neutron	Antineutron
Neutrino	Antineutrino

- Annihilation
 - Where a particle and a corresponding antiparticle meet
 - All their mass and KE is converted into two **gamma** photons of **equal frequency** moving in **opposite directions** (to conserve momentum)
 - Energy and momentum is conserved in the process
 - Minimum energy of a single photon = rest energy of the particle



- Pair production
 - A photon is converted into a **particle and a corresponding antiparticle**
 - Can only occur when the photon has an energy greater than the total rest energy of both particles
 - Minimum energy of a photon needed: $hf_{\min} = 2 \times \text{rest energy} = 2E_0$
 - Excess energy is converted into KE of particles



3.2.1.4 Particle interactions

- Four fundamental interactions

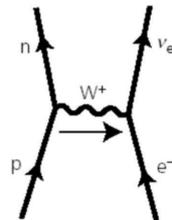
Interaction	Exchange particle /	Range (m)	Acts on	Strength
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	gauge bosons			
Strong nuclear	Pions (particles) Gluon (quarks)	10^{-15}	Hadrons	1st
Weak nuclear	W boson (W^+ or W^-)	10^{-18}	All particles	3rd
Electromagnetic	Virtual photon (γ)	Infinite	Charged particles	2nd
Gravity	Graviton	Infinite	Particles with mass	4th

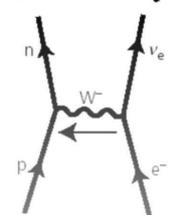
- Exchange particles
 - The **force carrier** for **fundamental forces**
 - Moves between the particles affected by the force and create the forces
 - Transfer energy, force, momentum, and (sometimes) charge** between the particles experiencing the force
 - Each fundamental force has its own exchange particles
- The weak nuclear force
 - Responsible for beta decay, electron capture and electron-proton collisions
 - Exchange particles = W bosons (W^+ or W^-)
 - Non-zero rest mass
 - Very short range ≤ 0.001 fm (10^{-18} m)
 - Positively or negatively charged
- Weak interactions

- Electron capture (electron-proton collisions)
 - Same equation + different exchange particle
 - $p + e^- \rightarrow n + \nu_e$

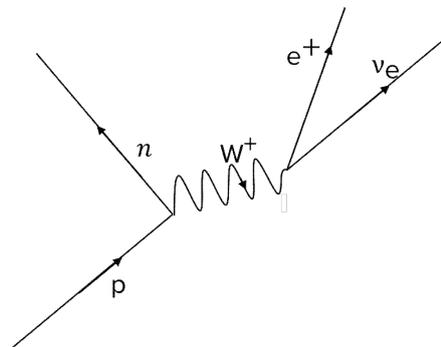
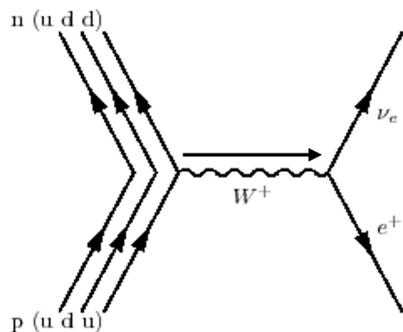
Electron capture
 $p + e^- \rightarrow n + \nu_e$



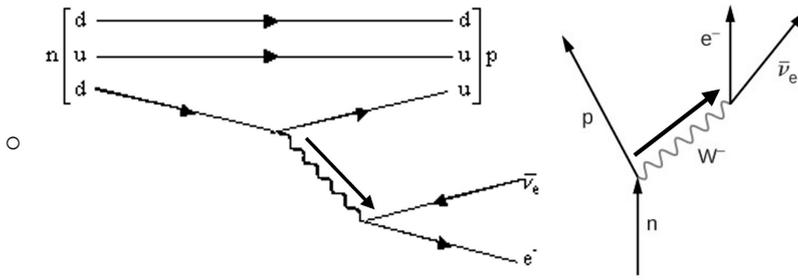
- Proton – electron collision
 $p + e^- \rightarrow n + \nu_e$



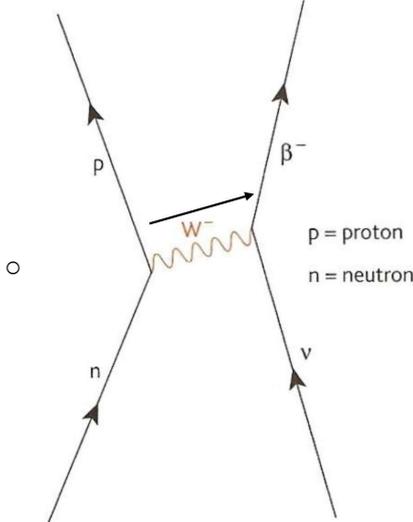
- Beta-plus decay
 $p \rightarrow n + e^+ + \nu_e$



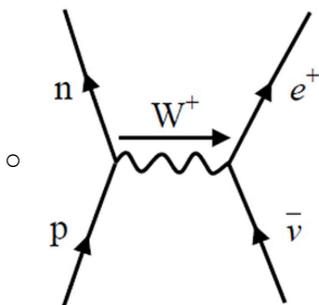
- Beta-minus decay
 $n \rightarrow p + e^- + \bar{\nu}_e$



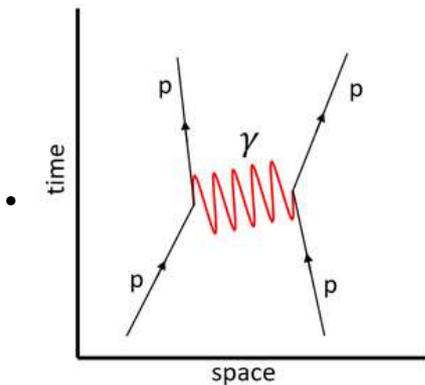
- Neutron-neutrino interaction



- Proton-antineutrino interaction



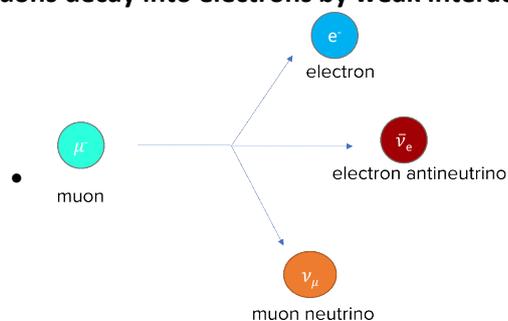
- Electrostatic interactions



3.2.1.5 Classification of particles

- Deducing whether is strong interaction or not
 - Leptons present: must be weak
 - Leptons not present: strong or weak
 - Strangeness not conserved: must be weak
- Classifying particles
 - All particles are either hadrons or leptons
 - Leptons
 - Fundamental particles - cannot be broken down any further
 - **Interacts via weak interaction only**

- Hadrons
 - Formed of quarks (fundamental particles)
 - **Interact through weak or strong interaction**
- Both experiences gravitational interaction and electromagnetic interaction (if charged)
- Types of hadrons
 - Baryons / antibaryons
 - Mesons
- Baryons / antibaryons
 - Hadrons that are formed of **3 quarks** (3 antiquarks for antibaryons)
 - **Proton is the only stable baryon**
 - All other baryons **eventually decay into protons**
 - (Neutrons are also baryons)
- Mesons
 - Formed of **1 quark + 1 antiquark**
 - Hadrons that **do not include protons** in their decay products
 - Very short life time (**annihilate almost immediately**) + **all unstable**
 - Pion / π meson
 - The lightest and most stable meson
 - Produced in high energy particle collisions, discovered in cosmic rays
 - Exchange particle for SNF
 - Different charges: π^+ , π^- , π^0
 - Kaon / K meson
 - Heavier + less stable
 - Produced by the strong interaction between pions and protons
 - Eventually decay into pions (many possibilities)
 - Different charges: K^+ , K^0 , K^-
- Baryon number
 - **1 = baryon / -1 = antibaryon / 0 = not a baryon**
 - A quantum number
 - **Always conserved** in particle interactions
- Types of leptons
 - Electron (e^-)
 - Relative charge = -1
 - **Electrons and their neutrinos / antiparticles are the only stable leptons**
 - Positron (e^+)
 - Will eventually encounter an electron and annihilate
 - Muon (μ^-)
 - Heavier than electrons
 - More unstable
 - Relative charge = -1
 - **Muons decay into electrons by weak interaction: $\mu^- \rightarrow e^- + \bar{\nu}_e + \nu_\mu$**



- Antimuon (μ^+)
 - **Antimuon (μ^+) decays into positrons: $\mu^+ \rightarrow e^+ + \nu_e + \bar{\nu}_\mu$**
- Neutrinos (ν_e, ν_μ)
 - Negligible mass & 0 charge
 - **The most abundant leptons in the universe**
- Tauons and & their neutrinos (not required to know)

- Lepton number
 - Gives the number of leptons
 - 1 = lepton, -1 = antilepton, 0 = not a lepton
 - **Conserved during reactions**
- Strangeness
 - A quantum number
 - Reflect the fact that strange particles are always created in pairs
 - Explain why some particle interactions take place more slowly than others / do not occur at all
 - **Always conserved in strong interactions**
 - **Change by 0, +1 or -1 in weak interactions**
- Strange particles
 - Particles which are **produced by the strong nuclear interaction but decay by the weak interaction**
 - Strange particles are **created in twos**
 - e.g. kaons (decay into pions through the weak interaction)
 - Assume all others are non-strange particles
 - **Particles with strange quarks have long lifetimes**
- Investigating particle physics
 - Particle accelerators may be built
 - These are very expensive + produce huge amounts of data
 - Scientific investigations rely on collaboration of scientists internationally
- Formulae sheet data (no need to memorise)

Class	Name	Symbol	Rest energy/MeV
photon	photon	γ	0
lepton	neutrino	ν_e	0
		ν_μ	0
	electron	e^\pm	0.510999
	muon	μ^\pm	105.659
mesons	π meson	π^\pm	139.576
		π^0	134.972
	K meson	K^\pm	493.821
		K^0	497.762
baryons	proton	p	938.257
	neutron	n	939.551

		Lepton number
Particles:	$e^-, \nu_e; \mu^-, \nu_\mu$	+ 1
Antiparticles:	$e^+, \bar{\nu}_e, \mu^+, \bar{\nu}_\mu$	- 1

3.2.1.6 Quarks and antiquarks

- Quarks
 - Baryons and mesons are composed of quarks
 - Quarks feel the **strong force**
 - Quarks are always found in pairs or triplets
- Combination of quarks and antiquarks in baryons / antibaryons

Particle	Combination	Baryon number	Strangeness
p	uud	1	0
n	udd	1	0
Antiproton	$\bar{u}\bar{u}\bar{d}$	-1	0

Antineutron	$\bar{u}\bar{d}\bar{d}$	-1	0
Σ^+	uus	1	-1
Σ^0	uds	1	-1
Σ^-	dds	1	-1

- Combination of quarks and antiquarks in mesons

Particle	Combination	Charge (e)	Strangeness	Baryon number
π^0	$u\bar{u}$ or $d\bar{d}$	0	0	0
π^+	$u\bar{d}$	+1	0	0
π^-	$\bar{u}d$	-1	0	0
K^0	$d\bar{s}$ or $\bar{d}s$	0	$d\bar{s} = +1, \bar{d}s = -1$	0
K^+	$u\bar{s}$	+1	+1	0
K^-	$\bar{u}s$	-1	-1	0

- Strangeness of particles

Strangeness	Particles
-3	Ω^-
-2	Ξ^-, Ξ^0
-1	$\Lambda, K^-, K^0, \Sigma^+, \Sigma^-, \Sigma^0$
0	$p, n, \pi^+, \pi^-, \pi^0$
1	K^+, K^0

- Neutron decay
 - Decay into proton as neutrons are baryons
 - A down quark changes to an up quark
 - $n \rightarrow p + e^- + \bar{\nu}_e$

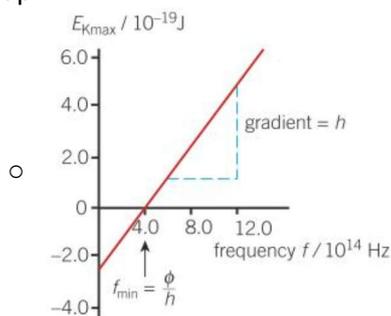
3.2.1.7 Applications of conservation laws

- Properties conserved in particle interactions
 - Energy and momentum: always
 - Reactants rest energy < products rest energy = reactants KE > products KE
 - Charge: always
 - Baryon number: always
 - Electron lepton number: always
 - Muon lepton number: always
 - Strangeness: only in strong interactions
 - All conservation laws obeyed = the interaction is possible
- β decay
 - β^- decay
 - A neutron in a neutron-rich nucleus will decay into a proton
 - A down quark changes to an up quark
 - $n \rightarrow p + e^- + \bar{\nu}_e$
 - β^+ decay
 - A proton in a proton-rich nucleus changes into a neutron
 - An up quark changes to a down quark
 - $p \rightarrow n + e^+ + \nu_e$

3.2.2 Electromagnetic radiation and quantum phenomena

3.2.2.1 The photoelectric effect

- The photoelectric effect
 - Photoelectrons are emitted from the surface of a metal after light above a certain frequency (threshold frequency) is shone on it
- Work function
 - The minimum energy to remove an electron from the metal surface when the metal is at **zero potential**
 - Denoted by ϕ
- Stopping potential (V_s)
 - The PD needed to apply across the metal to stop the photoelectrons with the maximum KE ($E_{k(\max)}$)
 - Minimum energy needed per unit charge to stop photoelectric emissions
 - $E_{k(\max)} = e \times V_s$
- Threshold frequency
 - The minimum frequency of the radiation / light / photon needed to liberate an electron from the surface of a material
 - $hf > \phi$
 - $f_{\min} = \frac{\phi}{h}$
- Why wave theory doesn't work
 - There is no photoemission below the threshold frequency even with bright light
 - Wave theory would allow gradual accumulation of energy to cause emission
 - Any frequency of light should be able to cause electron emission
 - Electrons are emitted with no noticeable decay
 - In wave theory time would elapse while an electron gains sufficient energy to leave the surface
 - Intensity of the light does not affect the KE of the emitted electrons
 - High intensity waves would be expected to give higher KE to an electron
- Explanation with the photon model
 - When light is incident on a metal surface an electron at the surface absorbs a **single photon** from the incident light and gains energy equal to hf
 - An electron can leave the metal surface if the energy gained $>$ the work function of the metal
 - Excess energy gained becomes KE of the photoelectron
- Effect of increasing the intensity of light
 - There are more photons striking the surface per second
 - Current increases as the number of electrons emitted per second increases
- Photoelectric equation
 - $E = hf = \phi + E_{k(\max)}$
 - $E_{k(\max)} = hf - \phi$
 - Graph



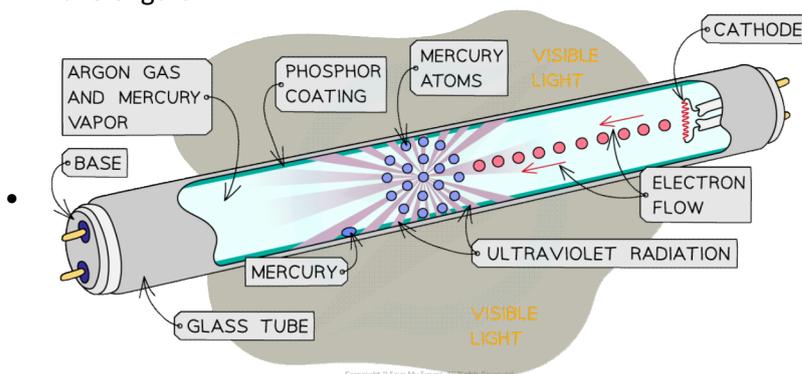
- Gradient = h
- y-intercept = $-\phi$

- Energy level of emitted electrons

- There exists a maximum value of energy
- Energy of photons are constant ($E = hf$)
- **One to one interaction** between photon and electron so a fixed amount of KE is transferred
- The energy required to remove an electron varies so the KE of electrons varies
 - Max KE = photon energy - work function
 - Deeper electrons require more energy to remove than ϕ

3.2.2.2 Collisions of electrons with atoms

- Electron energy level
 - Electrons in atoms can only exist in **discrete energy levels**
 - These electrons can gain energy from collisions with free electrons / absorbing photons
- Excitation
 - Electrons move up in energy level
 - It will quickly return to its original energy level (the ground state) and release energy gained as photons
- Ionisation
 - Electrons gain enough energy to be removed from the atom entirely
 - Occurs if **the energy of the free electron is greater than the ionisation energy**
- Excitation energies
 - The energy values at which an atom absorbs energy
- Fluorescent tube
 - Filled with mercury vapour
 - High voltage applied which accelerates free electrons through the tube
 - Free electrons collide with the mercury atoms
 - Electrons in the mercury atoms are raised to a higher energy level
 - The mercury atom become ionised → release more free electrons
 - The new free electrons collide with the mercury atoms, causing them to become excited
 - Mercury atoms **de-excite** and relaxes to a lower energy level
 - They release photons of energy equal to the energy difference between the levels
 - Frequency is mostly in the **UV range**
 - The fluorescent coating on the inside of the tube absorbs these UV photons and therefore electrons in the atoms of the coating become excited and de-excite releasing photons of **visible light**
 - Emitted radiation consists of (a range of) lower photon energies / frequencies or longer wavelengths

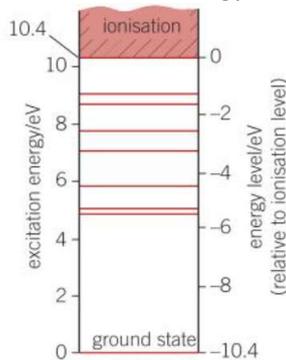


- Why electrons only need a minimum energy level to excite an atom
 - An exact amount of energy is needed to excite an atom to a certain energy level
 - All of the photon's energy will be absorbed in a **1 to 1 interaction**
 - Electron can transfer part of its energy to cause the excitation and continue moving at a lower KE

3.2.2.3 Energy levels and photon emission

- Ground state
 - When electrons / atoms are in there **lowest energy state** / most stable state
- Excited state

- **Electron** (in ground state) has moved to higher energy level / shell
- Ionisation energy
 - The minimum energy to remove an electron from an atom from the **ground state**
- De-excitation
 - The electron configuration in an excited atom is unstable due to a vacancy in the shell that the excited electron left
 - The vacancy is filled by an electron from an outer shell transferring to it
- Possible energy level of atoms
 - An atom can **only have certain levels** of energy
 - Each allowed energy level = a certain electron configuration of the atom



- Line spectrum
 - Obtained by passing the light from a fluorescent tube through a diffraction grating or prism
 - Each line = a specific wavelength of light emitted by the tube = corresponds a specific photon energy emitted
 - Show that electrons in atoms can only transition between discrete energy levels

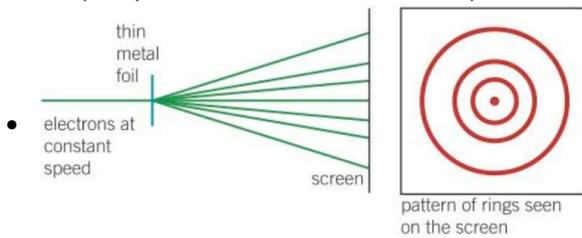


- Line absorption spectrum
 - Continuous spectrum with black lines at certain intervals
 - Obtained by passing white light through a cooled gas
 - Black lines represent the possible differences in energy levels
 - The atoms in the gas can only absorb photons of an energy equal to the exact difference between two energy levels
- Why only certain frequencies of light can be absorbed
 - Electrons occupy discrete energy levels
 - They need to absorb an exact amount of energy to move to a higher level
 - Photons need to have certain frequency to provide this energy ($E = hf$)
 - Energy required is the same for a particular atom
 - All energy of the photon is absorbed in **1 to 1** interaction between photon + electron
- Energy level difference
 - Difference between two energy levels = a line in the line spectrum = a specific photon energy emitted by a fluorescent tube / absorbed in a line absorption spectrum
 - Energy of photon emitted = energy lost by the electron = energy lost by the atom
 - Energy of the emitted photon $hf = E_1 - E_2$

3.2.2.4 Wave-particle duality

- Evidence for wave-particle duality of light / EM waves
 - Acting as wave: diffraction and interference
 - Acting as particle: photoelectric effect
- De Broglie hypothesis
 - Matter particles have a dual wave-particle nature
- Evidence for de Broglie hypothesis
 - Collisions by incident electrons move electrons in atoms between energy levels
 - Photon emitted when atoms de-excite or electrons move to lower energy levels
 - Wave properties of electrons

- Diffraction of electrons by a metal crystal
- Electrons can be **diffracted**, shown as concentric rings on screen
- Foil / graphite causes electrons to travel in particular directions
- Bright rings / maximum intensity occurs where waves arrive **in phase** and **interfere constructively**
- Particle behaviour would only produce a circle of light as particles scatter randomly
- **Only waves can experience diffraction** → electrons also have a dual wave-particle nature
- Similar to diffracting grating maxima when $n\lambda = d \sin \theta$
- Particle properties of electrons
 - Electrons must provide enough kinetic energy for light to be emitted
 - Instant light as electron can provide the energy in **discrete amounts** to excite the atoms
 - Waves → energy will accumulate gradually so time is needed until light is emitted & light will always be emitted no matter how low the energy is
- Property later also shown for other particles

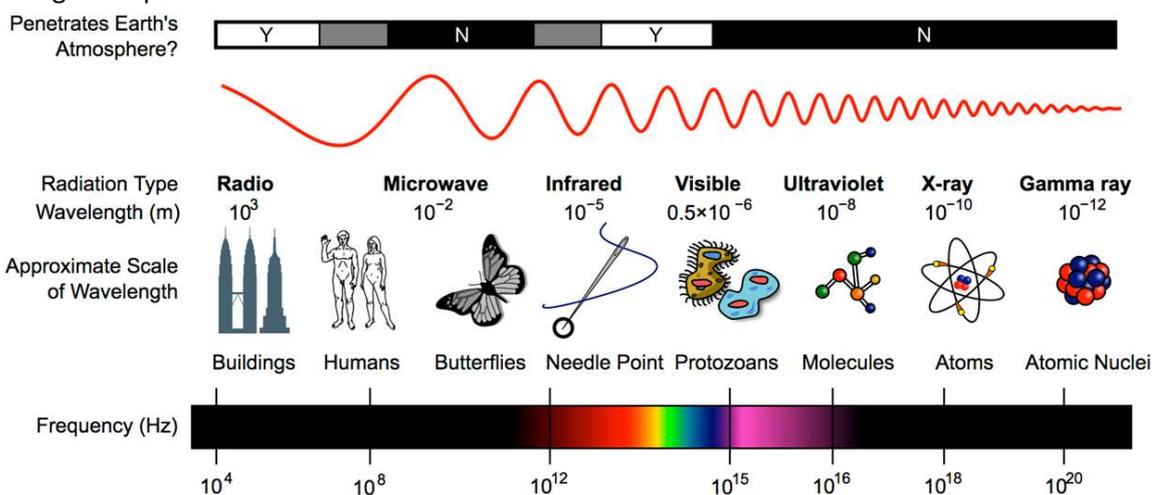


- De Broglie wavelength
 - The wavelength of the wave-like behaviour of a matter particle
 - $\lambda = \frac{h}{p} = \frac{h}{mv}$
 - Higher particle momentum = shorter wavelength = less diffraction = concentric rings of the interference pattern become closer
- Change in understanding of matter
 - Knowledge and understanding of the nature of matter changes over time in line with new experimental evidence gathered
 - Such changes need to be evaluated through peer review and validated by the scientific community before being accepted

3.3.1 Progressive and stationary waves

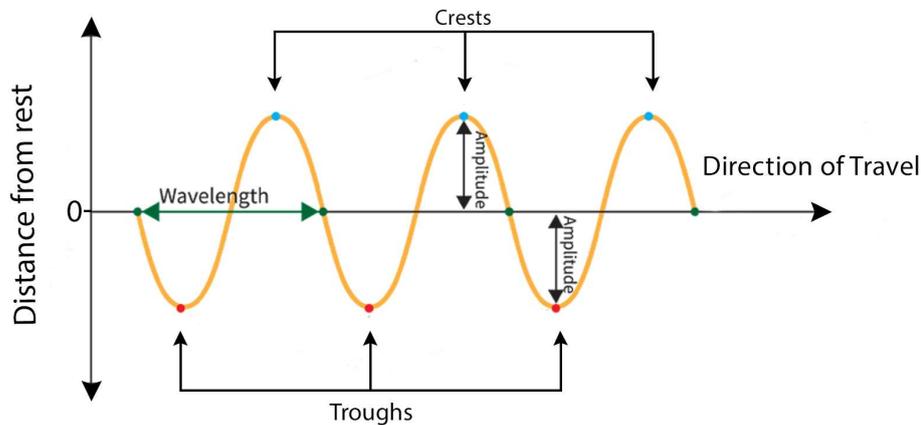
3.3.1.1 Progressive waves

- Mechanical waves
 - Involve particles in a substance vibrating
- Electromagnetic waves
 - Travel through space without the need for a medium
- Electromagnetic spectrum



- Progressive wave
 - A wave that transfers energy and momentum from one point to another without transferring the medium itself
 - Made up of particles of an oscillating medium
- Terminologies

Term	Definition
Displacement	The vibrating particle's distance and direction from its equilibrium position
Amplitude	A wave's maximum displacement from the equilibrium position (unit = m)
Frequency f	The number of complete oscillations passing through a point per second (unit = Hz)
Period T	The time taken to make one oscillation (unit = s)
Wavelength λ	The length of one whole oscillation (e.g. the distance between successive peaks/troughs) (unit = m)
Speed c	Distance travelled by the wave per unit time (unit = ms^{-1})
Phase	The fraction of a cycle a vibrating particle has completed since the start of the cycle
Cycle	One complete cycle of a wave is from maximum displacement to next maximum displacement



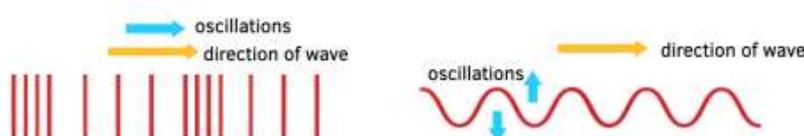
- Phase difference
 - The fraction of a cycle between the vibration of two particles
 - phase difference in radians = $\frac{2\pi d}{\lambda} = \frac{2\pi \times \text{distance between two points}}{\text{wavelength}}$
- In phase
 - Two points on a wave are in phase if they are both at the same point of the wave cycle
 - Same displacement and velocity
 - Phase difference is a multiple of $360^\circ / 2\pi$
- Completely out of phase / in anti-phase
 - $(2n + 1)\pi$ apart in phase
- Wave speed
 - $c = f\lambda$
- Frequency / period conversion
 - $f = \frac{1}{T}$
 - $T = \frac{1}{f}$
- Properties of waves
 - Reflection
 - Refraction
 - Diffraction

3.3.1.2 Longitudinal and transverse waves

- Longitudinal and transverse waves
 - Transverse waves
 - The particles oscillate **perpendicular** to the direction of travel of the wave
 - **Can be polarised**
 - e.g. EM waves, waves on a string
 - Longitudinal waves
 - The particles oscillate **parallel** to the direction of travel of wave
 - The particles get compressed so they have more energy than the particles around them
 - When they vibrate they transfer energy to particles nearby → more compressions
 - **Cannot be polarised**
 - Cannot travel in vacuums (require a medium to propagate)
 - e.g. sound waves

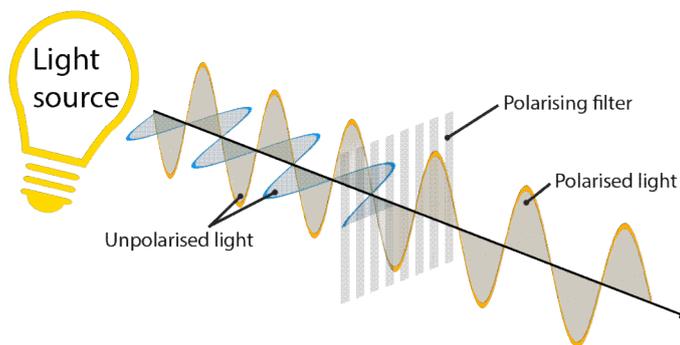
Longitudinal Waves

Transverse Waves



- Types of waves
 - Mechanical waves
 - Oscillations of the particles of the medium

- Electromagnetic waves
 - Oscillating electric and magnetic field that progress through space without the need for a substance
 - Transverse waves
 - All have the same speed in vacuum ($3 \times 10^8 \text{ ms}^{-1}$)
- Polarisation
 - Can only happen for transverse waves
 - Oscillations of particles / fields are perpendicular to the direction of energy propagation
 - Wave's oscillations exist in more than one plane initially
 - Particle oscillations **restricted to a single plane** perpendicular to the direction of wave propagation
 - Cannot occur on longitudinal waves as it does not oscillate perpendicular to the direction of travel
 - (Transverse waves are called plane-polarised if the vibrations occur in one plane only, more than one plane = unpolarised)
- Polarising filter

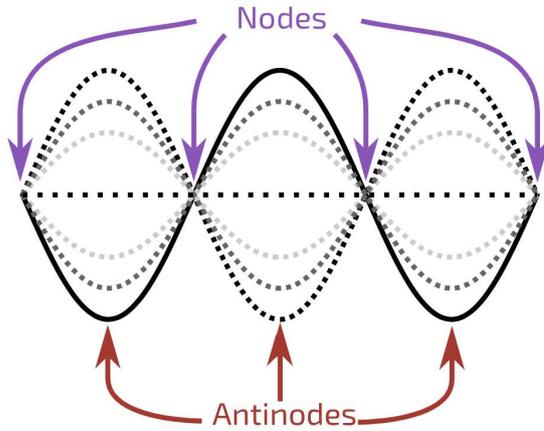


- Only the wave oscillating along the transmission axis can completely pass through
- Greater angle between wave + axis = lower light intensity
- Waves oscillating perpendicular to the transmission axis cannot pass through = 0 intensity
- Why only transverse waves can be polarised
 - Transverse waves oscillate perpendicular to direction of travel of wave
 - They initially oscillate in many different planes
 - Intensity is reduced due to oscillations being limited to one plane only
 - Longitudinal waves oscillate parallel to direction of travel of wave
 - There is no perpendicular plane to restrict the oscillations to
- Applications of polarisation
 - Polaroid sunglasses
 - **Vertical** transmission axis
 - Reduce glare by blocking (nearly all) **horizontally polarised light** reflected from water and glass
 - Only oscillations in the plane of the filter is allowed, other directions absorbed
 - Only transmits the vertical component of the unpolarised light from objects
 - **50%** of the original light passes through
 - Reduces light from surface reflection more than light from objects so it is easier to see
 - EV and radio signals
 - Plane-polarised by the orientation of the rods on the transmitting aerial
 - The receiving aerial must be aligned in the same plane of polarisation to receive the signal at full strength

3.3.1.3 Principle of superposition of waves and formation of stationary waves

- The principle of superposition
 - When two or more waves arrive at one point, the resultant displacement is the sum of the displacement of each wave
- Constructive interference
 - Occurs when 2 waves have displacement in the same direction (arrives in phase)

- Destructive interference
 - Occurs when one wave has positive displacement and the other has negative displacement (arrives out of phase)
 - If the waves have equal but opposite displacements (π rad out of phase), total destructive interference occurs (zero amplitude)
- Stationary waves
 - Waves where there is no net transfer of energy and momentum from one point to another



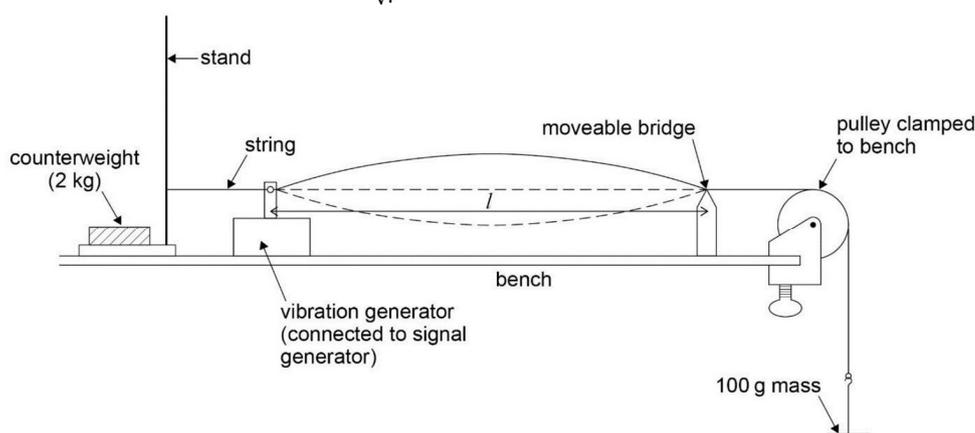
- Formation of stationary waves
 - Formed by the **superposition** of two or more **progressive waves** of the **same frequency and wavelength** and **similar amplitudes** passing through each other in **opposite directions in the same medium**
 - (The waves are emitted by ..., **reflected through 180°** by ...)
 - Amplitudes of the two waves do not need to be the same
 - Constructive interference occurs at where the waves meet in phase so antinodes are formed
 - Destructive interference occurs at where the waves meet completely out of phase so nodes are formed
- Nodes
 - Fixed points in a stationary wave where the amplitude is **minimum** (usually zero)
 - Distance between 2 nodes = $\frac{\text{wavelength}}{2}$
- Antinode
 - Fixed point in a stationary wave pattern where the amplitude is **maximum**
 - The particles have **maximum energy** at the antinode
- Progressive waves + stationary waves comparison

	Stationary	Progressive
Energy & momentum	No net transfer of energy from one point to another through space	Energy and momentum is transferred from 1 point to another
Wavelength	Wavelength = 2 × distance between adjacent nodes	Wavelength = distance between 2 particles at the same phase
Frequency	All particles except the particles at the nodes vibrate at the same frequency	All particles vibrate at the same frequency
Amplitude	The amplitude varies from minimum (0) at the nodes to maximum at the antinode	The amplitude is the same for each point along wave
Phase difference between 2 particles (rad)	Particles immediately on either side of a node are moving in opposite directions (180° out of phase) Between adjacent nodes all particles are vibrate in phase phase difference = $m\pi$ = number of nodes between 2 particles × π	phase difference = $\frac{2\pi d}{\lambda}$ Adjacent points vibrate with different phase

- Examples of stationary waves
 - Transverse stationary waves
 - String fixed at one end and the other end is fixed to a driving oscillator / plucked
 - Wave reflected at the fixed end of the string by 180°
 - Two waves superpose with each other
 - Both ends are fixed so both ends of the string are always nodes
 - Stationary microwaves
 - Reflected on a soft surface
 - **The reflected end is an antinode, the emitter end is a node**
 - A microwave probe can be used to find the nodes and antinodes
 - Longitudinal stationary waves
 - Sound waves
 - Speaker causes the wave → **antinode**
 - Open end: when air leaves the tube the pressure around it is lower so it expands → air pushed back to the tube → **antinode**
 - Close end: reflects the wave which reverses its displacement → cancelled out by upcoming wave → **node**
- Harmonics
 - The number of **antinodes** on the string
- First harmonic frequency / fundamental frequency
 - The lowest frequency at which a stationary wave forms
 - Forms a stationary wave with two nodes and a single antinode
 - Distance between adjacent nodes = half a wavelength
 - $\lambda = 2L$
 - $f_0 = \frac{c}{2L} = \frac{1}{2L} \sqrt{\frac{T}{\mu}}$ (T = tension in the wire, $\mu = \frac{m}{L}$ = mass per unit length)
 - n th harmonic frequency = $n \times$ first harmonic frequency = nf_1
 - n th harmonic frequency = nodes at a distance of $\frac{L}{n}$
- Factors affecting the fundamental frequency
 - Mass per unit length
 - Tension
 - Length
 - Temperature

Required Practical 1 - Stationary Waves

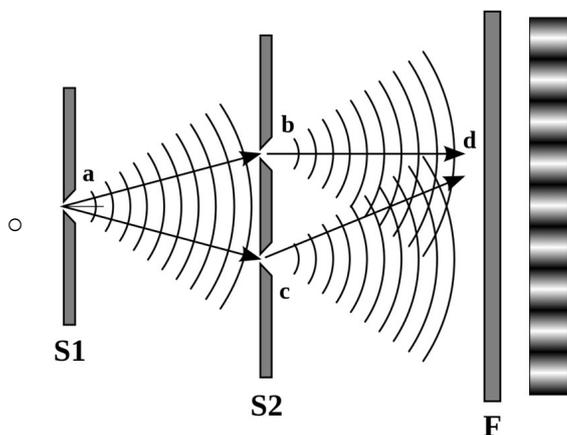
- Frequency vs length / tension / mass per unit length
 - Keep other variables constant
 - Use a signal generator + vibrator connected to signal generator to produce the vibrations
 - Measure length using ruler / tension by hanging mass at one end / mass per unit length by changing the wire
 - Graph of f vs. $\frac{1}{l}$ / f vs \sqrt{T} / f vs $\frac{1}{\sqrt{\mu}}$

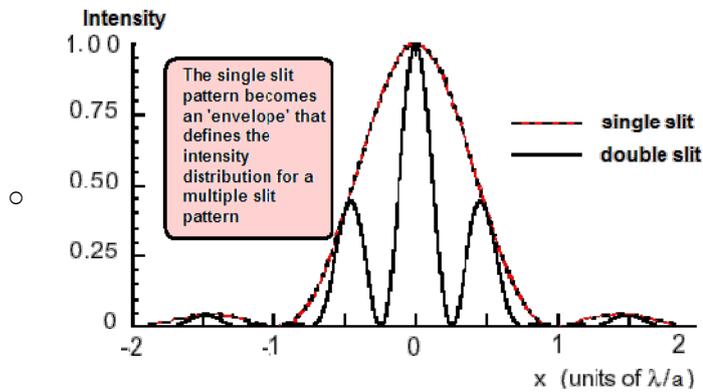


3.3.2 Refraction, diffraction and interference

3.3.2.1 Interference

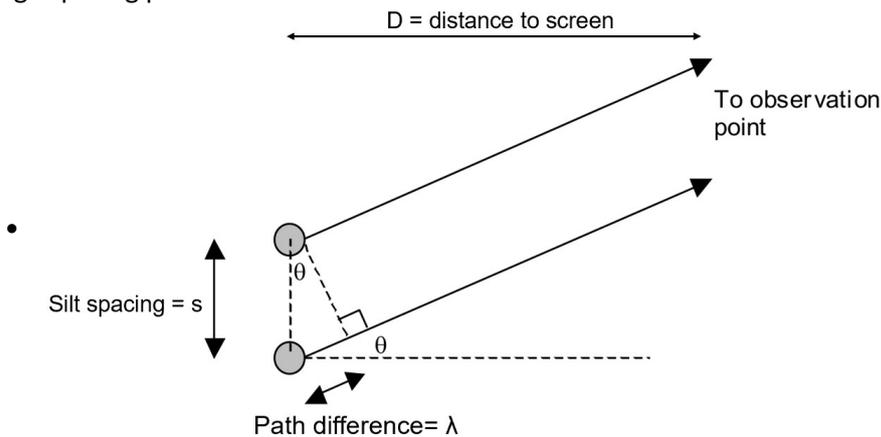
- Coherence
 - Waves with a **constant phase difference** and the **same frequency and wavelength**
- Monochromatic
 - Light waves with a **single wavelength only**
- Lasers
 - **Coherent and monochromatic**
 - Usually used as sources of light in diffraction experiments as they form clear interference patterns
- Path difference
 - The difference in the distance travelled by two waves from their sources to where they meet
- Interference of monochromatic light
 - Path difference = $n\lambda$ → constructive interference, gives maximum intensity / reinforcement
 - Path difference = $(n + \frac{1}{2})\lambda$ → destructive interference, gives 0 intensity / cancellation
- Interference of longitudinal waves (sound waves)
 - Constructive interference / reinforcement
 - Compression + compression / rarefaction + rarefaction → greater volume
 - Destructive interference / cancellation
 - Compression + rarefaction → 0 volume, used for noise cancellation
- Young's double slit experiment
 - Condition for light source
 - Monochromatic light source → use colour filter
 - Coherent → single slit between light source and double slit
 - Both slits will be illuminated from the same source so they receive light of the same wavelength
 - Paths to both slits are of constant length giving constant phase difference (normally in phase)
 - Laser is both monochromatic + coherent so no colour filter / single slit needed
 - Procedure
 - Shine a coherent light source through 2 slits about the same size as the wavelength of laser light so the light diffracts / use 2 coherent sources
 - Each slit acts as a **coherent point source** making a pattern of light and dark fringes
 - Light fringes are formed where the light from both slits meet **in phase** and **interferes constructively** (path difference = $n\lambda$)
 - Dark fringes are formed where the light from both slits meets **completely out of phase** and **interferes destructively** (path difference = $(n + \frac{1}{2})\lambda$)





- (Bright) Fringe spacing
 - $w = \frac{\lambda D}{s} = \frac{\text{wavelength} \times \text{distance between slit and screen}}{\text{slit spacing}}$

- Fringe spacing proof

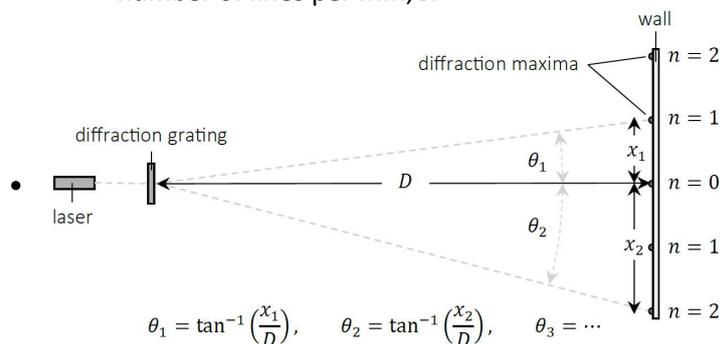


- $\sin \theta = \frac{\lambda}{s}$
- $\tan \theta = \frac{w}{D}$
- When θ is small: $\sin \theta \approx \tan \theta \approx \theta$ so $\frac{\lambda}{s} = \frac{w}{D}$
- Hence $w = \frac{\lambda D}{s}$
- Significance of Young's double slit experiment
 - **Proved the wave nature of light** since diffraction and interference are wave properties
 - Proved that EM radiation must act as a wave
 - Disproved the theories that light is formed of tiny particles
 - Knowledge and understanding of any scientific concept changes over time in accordance to the experimental evidence gathered by the scientific community
- Interference pattern with white light
 - Wider maxima
 - Less intense diffraction pattern with a central white fringe (all colours are present)
 - Alternating bright fringes which are spectra, violet is the closest to the central maximum and red is the furthest
- Safety precautions with lasers
 - Do not look directly at a laser beam even when it is reflected
 - Wear laser safety goggles
 - Don't shine the laser at reflective surfaces
 - Display a warning sign
 - Never shine the laser at a person

Required practical 2

- Determining slit separation for double slit
 - Illuminate a double slit with a red laser of known wavelength, λ
 - Project the interference pattern onto a white screen a distance, D , away from the slits
 - Measure D with a tape measure

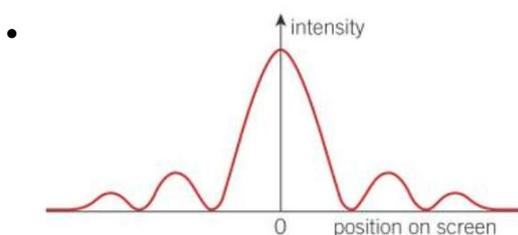
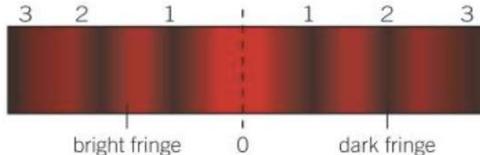
- The fringe spacing, w , can be measured by measuring across several visible fringes
 - Measure w using a meter rule
 - Find the **mean** fringe spacing
- Use the double slit formula, $w = \frac{\lambda D}{s}$ and the measurements to determine the slit separation, s .
- Finding average grating spacing
 - Illuminate a diffraction grating with a known grating spacing with a red laser of known wavelength, λ
 - Project the diffraction pattern onto a white screen at a distance D away from the grating.
 - Measure D with a tape measure
 - Measure the angles of diffraction, θ_n for multiple diffraction maxima
 - Measure distance from centre + use trigonometry
 - Plot a graph of $\sin \theta_n$ against n
 - gradient = $\frac{\lambda}{d}$
 - Use the gradient to determine the average grating spacing d and hence the average number of lines per mm, N



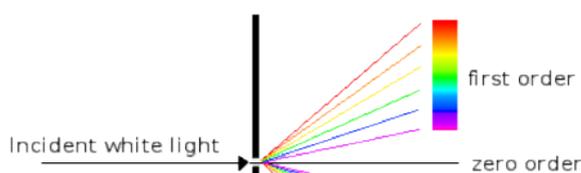
- Finding wavelength of light
 - Use a diffraction grating with a known average grating spacing and use the gradient to find the wavelength instead, otherwise same as above

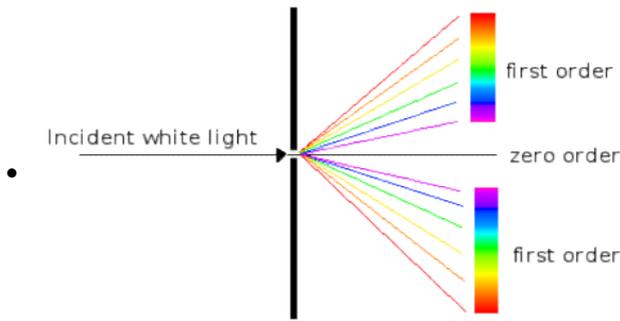
3.3.2.2 Diffraction

- Single slit diffraction with monochromatic light
 - Central maximum with highest intensity
 - Decreasing intensity fringes on both sides, equally spaced
 - Central fringe is **twice the width** of other fringes

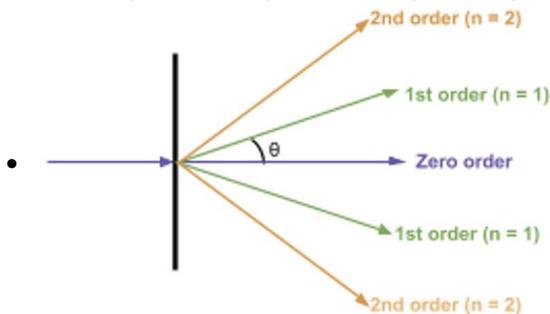


- Single slit diffraction with white (non-monochromatic) light
 - White central maximum
 - Distinct fringes shown with subsidiary maxima
 - Fringes on both sides show as spectrums
 - Red furthest, violet closest to the centre in subsidiary maxima





- Effect of using narrower slit / longer wavelength
 - Waves are more diffracted
 - Wider spacing
 - Lower intensity of fringes as the energy is spread over a larger area
- Fringe spacing for single slit diffraction
 - $w = \frac{\lambda \times \text{distance from slit to screen}}{\text{slit width}} = \frac{\lambda D}{a}$
- Diffraction grating explanation
 - A slide containing many equally spaced slits very close together
 - Light passing through each slit is **diffracted**
 - Light from different slits **superpose**
 - When the path difference between adjacent slits is a whole number of wavelengths the light waves arrive **in phase** and **constructive interference** occurs
 - Distances from the centre where maxima occur = $d \sin \theta = n\lambda$ (d = distance between slits)
 - Angle of diffraction between each transmitted beam and the central beam increases if light of a longer wavelength or a grating with closer slits is used



- Comparison to double slit
 - **Much sharper and brighter image** when monochromatic light is passed through
- White light incident
 - Spectrum is seen when white light is used
 - Different colours of light have different maxima positions
 - Line absorption spectra and line emission spectra can determine the elements in a substance (see photoelectric effect)
- Maximum number of orders visible
 - Use $\theta = 90^\circ$
 - $n_{max} = \left\lfloor \frac{d}{\lambda} \right\rfloor$
- Measuring wavelength of light
 - Diffraction patterns are measured using a spectrometer
 - Angles measured accurate to 1 arc minute ($\frac{1}{60}^\circ$)
 - It can be used to study light from any source and measure wavelengths very accurately
 - Angle measured using a known wavelength \rightarrow grating spacing calculated
 - Grating can then be used to measure the wavelength of any light
- Applications of diffraction grating
 - Diffraction gratings can be used to observe and measure spectral lines
 - Line emission spectra can be used to identify elements in the vapour gas of the vapour lamp (similar to line absorption spectra)

3.3.2.3 Refraction at a plane surface

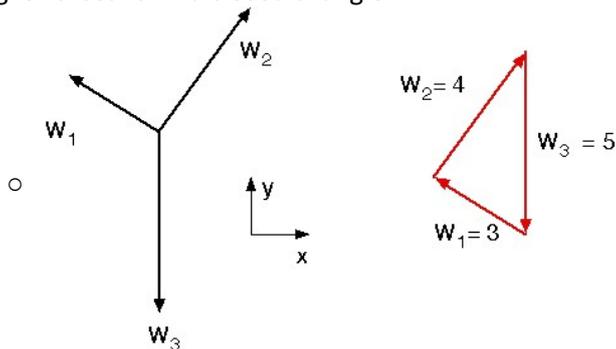
- Refraction
 - Change of direction and **wavelength** when a wave crosses a boundary and its speed changes
- Refractive index of a substance
 - $n = \frac{c}{c_s} = \frac{\sin i}{\sin r}$
 - Refractive index of air ≈ 1
 - Higher refractive index = light travels slower in the substance
- Refractive index of different colours
 - Longer wavelength = smaller refractive index = travels faster
 - Red has the smallest refractive index as it has the longest wavelength
 - Violet has the largest refractive index as it has the shortest wavelength
- Snell's law
 - $n_1 \sin \theta_1 = n_2 \sin \theta_2$
- Total internal reflection (TIR)
 - For ray going from **more dense** to **less dense** substance & the angle of incidence exceeds the critical angle
 - No refracted light wave since the angle of refraction $> 90^\circ$ so **all the light is reflected**
- Critical angle
 - The angle of incidence at which the angle of refraction is 90°
 - $\sin \theta_c = \frac{n_2}{n_1} (n_2 < n_1)$
- Pulse broadening
 - The length of a pulse is widened so it may overlap with the next pulse
 - Distorts the information in the final pulse
- Pulse absorption
 - Energy is absorbed by the fibre
 - Amplitude is reduced so information can be lost
- Solution to pulse absorption
 - Use more transparent core
 - Use pulse repeaters to regenerate the pulse before significant pulse broadening has taken place
- Material / spectral dispersion
 - Happens if white light is used instead of monochromatic light
 - Different wavelengths have different speeds due to different refractive indices within the core
 - Red light has the longest wavelength \rightarrow lowest refractive index, fastest
 - Violet light has the shortest wavelength \rightarrow highest refractive index, slowest
 - Causes **pulse broadening**
- Solution to material dispersion
 - Use monochromatic light
 - Use of shorter repeaters so that the pulse is reformed before significant pulse broadening has taken place
- Modal / multipath dispersion
 - Light waves entered at **different angles of incidence** so they are spread out
 - They travel different distances as they take different paths and arrive at the other end at different times
 - Causes **pulse broadening**
- Solution to modal dispersion
 - Use monomode fibre / narrower core
 - Use a cladding with its refractive index as close to the core as possible (larger critical angle \rightarrow less TIR)
- Optical fibre structure
 - Core
 - The transmission medium for EM waves to progress
 - Cladding
 - Protects the outer surface of the core from scratching which could lead to light leaving

- the core
 - Ensure that no light leaves the core
 - RI of cladding < RI of core
 - Maintains quality / reduces pulse broadening
 - Prevent crossover of signal to other fibres
- Refractive index of cladding
 - Similar RI between cladding and core
 - Larger critical angle → less TIR → less modal dispersion
 - RI of cladding much smaller than core
 - Smaller critical angle (greater acceptance angle) → less light escape → more light collected
- Types of optical fibre
 - Step index fibre
 - The refractive index of each component increases moving from the outside to the centre of the fibre
 - The refractive index within each component is **uniform**
 - Graded index fibre
 - Has a core that has a **gradually increasing** refractive index from outside to centre
- Applications of optical fibres
 - Endoscopes
 - Transmission of data for communications
- Producing coherent image
 - An incoherent bundle cannot be used to form an image because the ends of the individual fibres are arranged randomly so the image is incorrect
 - In a coherent bundle, the fibres have the same spatial position at each end of the bundle.
 - The light emitted from the end of the bundle is an **exact copy** of the incident light and a **single image** can be reproduced and analysed
 - Coherent bundles are expensive to manufacture so incoherent bundles are used for illumination
- Advantages of optical fibres
 - Less loss of strength
 - No interference
 - Greater bandwidth for more information per second
 - Increased security

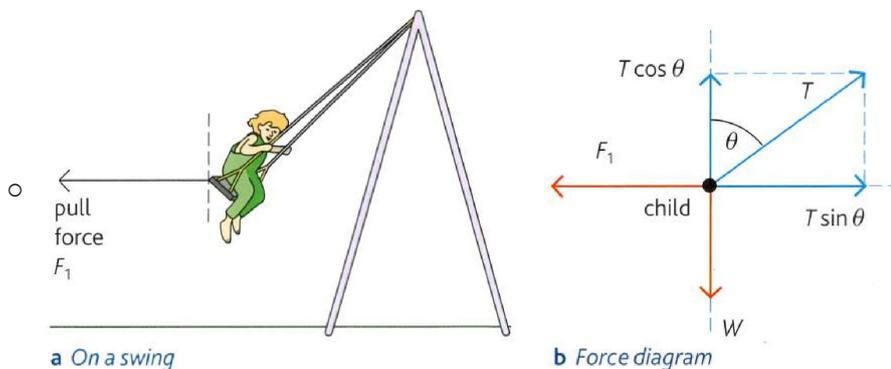
3.4.1 Force, energy and momentum

3.4.1.1 Scalars and vectors

- Vector
 - Any physical quantity that has a direction as well as a magnitude
 - e.g. velocity, force / weight, acceleration, displacement
- Scalar
 - Any physical quantity that is not directional
 - e.g. speed, mass, distance, temperature
- Conditions for equilibrium
 - For an object to be in equilibrium, the sum of all the forces acting on an it must be 0
 - e.g. 3 forces form a closed triangle



- Explaining what forces balance each other
 - e.g. child on swing
 - Pull force balances with the **horizontal component** of tension
 - Weight balances with the **vertical component** of tension



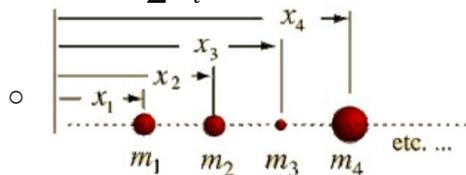
3.4.1.2 Moments

- Moment formula
 - Moment of a force about a point
= force \times perpendicular distance from the pivot point to the line of action of the force
- Couple
 - A pair of equal and opposite parallel / coplanar forces acting on a body along different points
 - Exerts a turning force on a body
 - Moment of couple
= force \times perpendicular distance between the lines of action of the forces
- Principle of moments
 - For an object in equilibrium, there is no resultant turning forces about any pivot
- Centre of mass
 - The point at which an object's mass / weight acts
 - The point through which a single force on the body has no turning effect
- Finding centre of mass
 - Uniform regular solid

- Centre of mass at the centre
- Non-regular card
 - Hang object (and plumb line) by first pivot
 - Draw first line vertically below pivot (by sketching a plumb line hang from the pivot)
 - Hang object (and the plumb line) by second pivot
 - Draw second line vertically below pivot
 - Intersection of lines is the centre of mass

- Multiple mass on a rod

- $$x_{cm} = \frac{\sum m_i x_i}{\sum m_i}$$

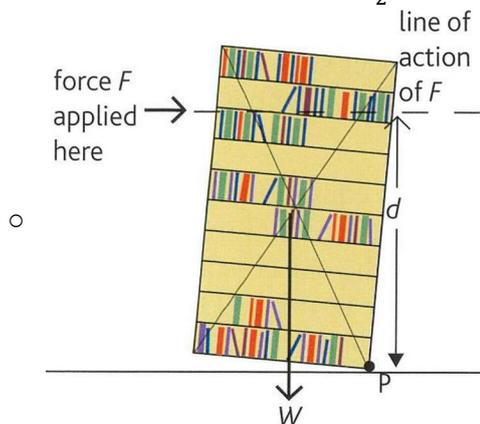


- Equilibrium stability

- Stable
 - Object returns to its equilibrium position if displaced (a little)
 - Wide base, low centre of gravity
 - Tilted for a certain angle before centre of mass crosses the pivot point and topple
- Unstable
 - Object does not return to its equilibrium position if displaced
 - Topple immediately after being tilted
- Neutral
 - Stay in place when left alone
 - Stay in the new position when moved
 - The object's centre of mass is always exactly over the point which is its 'base'

- Tilting / topping

- Tilting
 - An object resting on a surface is acted on by a force that raises it up on 1 side
 - For an object to tilt: $Fd > \frac{Wb}{2}$ (b = width of base)



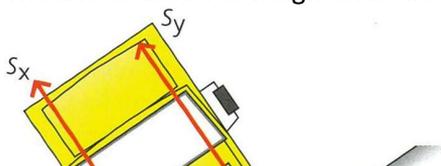
- Toppling
 - Tilted too far
 - **Line of action of its weight passes beyond the pivot**

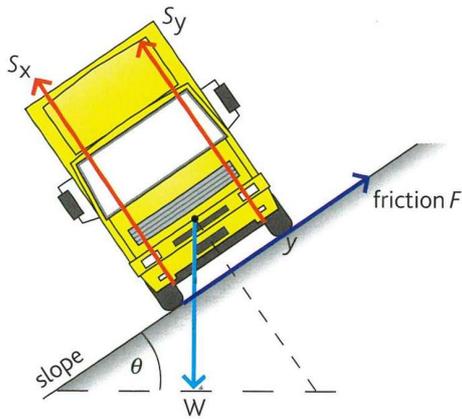
- Two support problems

- When an object is in equilibrium and supported by 2 points then the 2 supports add up to the weight of the object
- **The support closer to the centre of mass provides more of the support force**

- On a slope

- The line of action of weight must lie **inside the base** of the object to prevent tilting





- $S_x > S_y$ since x is lower than y (more moment is needed to be produced from x as it is closer to the centre of mass)
- Conditions for equilibrium
 - No resultant force
 - No resultant moment / torque (the principle of moments must apply)

3.4.1.3 Motion along a straight line

- Terms

Term	Definition
Speed	A scalar quantity describing how quickly an object is travelling
Displacement (s)	The overall distance travelled from the starting position (includes a direction, vector quantity)
Velocity (v)	Rate of change of displacement ($= \frac{\Delta s}{\Delta t}$)
• Instantaneous velocity	The velocity of an object at a specific point in time
Average velocity	The velocity of an object over a specified time frame
Acceleration (a)	Rate of change of velocity ($= \frac{\Delta v}{\Delta t}$)
Uniform acceleration	The acceleration of an object is constant

- SUVAT equations
 - For **uniform acceleration**
 - $v = u + at$
 - $s = \left(\frac{u + v}{2}\right)t$
 - $s = ut + \frac{1}{2}at^2$
 - $s = vt - \frac{1}{2}at^2$
 - $v^2 = u^2 + 2as$

- Motion graphs

	Displacement-time	Velocity-time	Acceleration-time
• Gradient	Velocity	Acceleration	/
Area	/	(Change in) displacement	Change in velocity

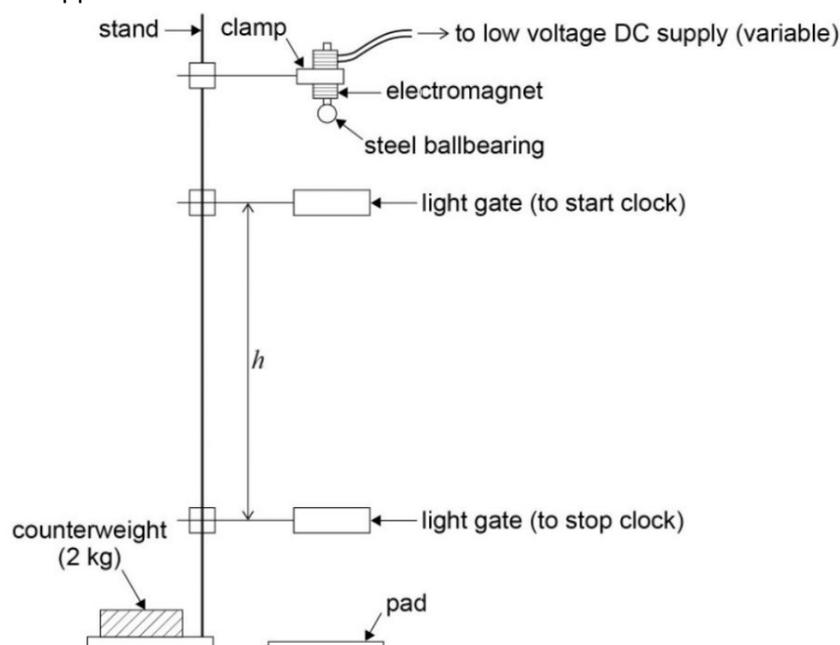
- Free fall
 - $u = 0$
 - $a = g$
- Light gate

- speed through the light gate = $\frac{\text{length of the object}}{\text{time for the light to be obscured}}$

Required practical 3 - determining g

- Equipment
 - Stand
 - Bosses and clamps
 - Electromagnet
 - Steel ball bearing
 - Light gate
 - Timer (connected to the light gate)
 - Soft cushion pad

- How to determine g by free fall
 - Set up the apparatus as shown

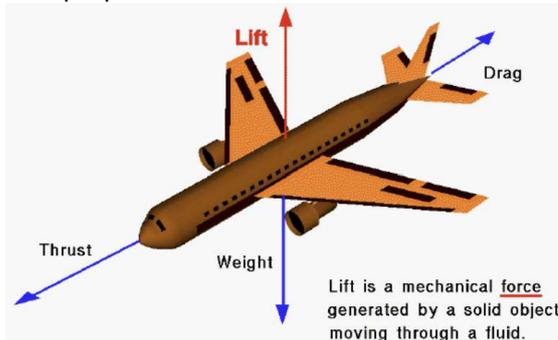


- The position of the lower light gate should be adjusted such that the height h is 0.500m, measured using the metre rule
- Turn on the electromagnet and attach the ball bearing
- Reset the timer to zero and switch off the electromagnet
- Read and record the time t on the timer for the ball to pass through the 2 light dates
- Reduce h by 0.050m by moving the **lower light gate** upwards and repeat this, reducing h by 0.050m each time until h reaches 0.250m (at least 5-10 values of h)
- Repeat the experiment twice more for each value of h and find and record the mean t for each h
- Plot a graph of $\frac{2h}{t}$ against t and draw a line of best fit ($\frac{2h}{t} = 2u + gt$)
- Gradient = g , y-intercept = $2u$
 - (You might want to draw lines of maximum and minimum gradient and find the mean gradient)
- Errors
 - Systematic
 - Residue magnetism after the electromagnet is switched off may cause t to be recorded as longer than it should be
 - Air resistance reduces the value of g determined
 - Random
 - Large uncertainty in h from using a metre rule with a precision of 1 mm
 - Parallax error from reading h
 - The ball may not fall accurately down the centre of each light gate (less time obscuring the light)
 - Random errors are reduced through repeating the experiment for each value of h at least 3-5 times and finding an average time, t
- Safety

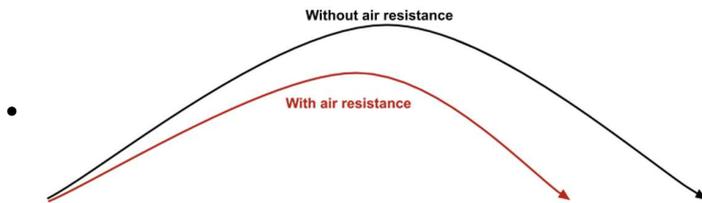
- The electromagnetic requires current
 - No water near it
 - Only switch on the current to the electromagnet once everything is set up to avoid electrocution
- A cushion or a soft surface must be used to catch the ball-bearing so it doesn't roll off / damage the surface
- The tall clamp stand needs to be attached to a surface with a G clamp so it stays rigid

3.4.1.4 Projectile motion

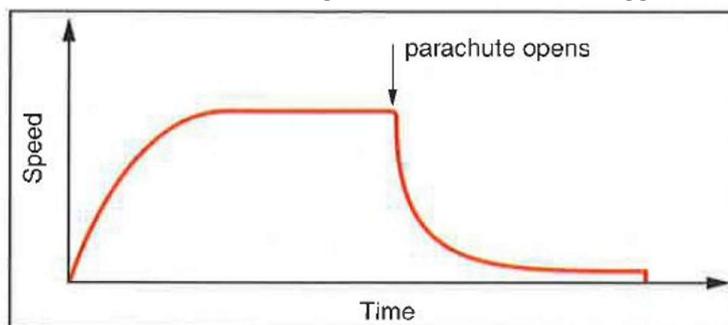
- Motion equations ignoring air resistance
 - $v_x = u \cos \theta$
 - $x = ut \cos \theta$
 - $v_y = u \sin \theta - gt$
 - $y = ut \sin \theta - \frac{1}{2}gt^2$
- Range and maximum height formula (not required but useful)
 - Maximum height = $\frac{u^2 \sin^2 \theta}{2g}$
 - Horizontal range = $\frac{u^2 \sin 2\theta}{g}$
 - Time to maximum height = $\frac{u \sin \theta}{g}$
 - Time back to starting height = $\frac{2u \sin \theta}{g}$
- Friction
 - A force which opposes the motion of an object
 - AKA drag / air resistance
 - Convert KE into other forms of energy such as heat and sound (work done on the surface / fluid)
- Lift
 - An upward force which acts on objects travelling in a fluid
 - Caused by the object creating a change in the direction of the fluid flow
 - Happens if the shape of the projectile causes the air to flow faster over the top of the object than underneath it
 - Pressure of air on the top surface < pressure of the air on the bottom surface
 - Produces a net upward force
 - Acts perpendicular to the direction of fluid flow



- Effect of air resistance (friction)
 - Air resistance / drag force acts in the opposite direction of motion of the projectile
 - Increases as the projectile's speed increases
 - Has both horizontal and vertical components
 - Reduces both the horizontal speed of the projectile and its range
 - Reduces the maximum height of the projectile if its initial direction is above the horizontal and makes its descent steeper than its ascent



- Terminal velocity
 - Occurs where **the frictional forces acting on an object and the driving forces are equal**
 - No resultant force → no acceleration → travels at constant speed / velocity
- Terminal velocity for objects falling
 - Start initially with free fall (uniform acceleration) briefly
 - The only force acting on the object is weight
 - (Other forces are very small and negligible)
 - Speed still increases but acceleration decreases
 - Air resistance increase because speed increase
 - Resultant force gets smaller
 - Eventually the object falls in uniform velocity (reached terminal velocity)
 - Weight balanced exactly by resistive force upwards
 - Resultant force = 0 so there is no acceleration
 - Air resistance is not increasing anymore because speed is not increasing
 - Potential energy of the object is transferred to the internal energy of the fluid by drag forces
 - Effect of parachute
 - Increase air resistance due to larger area perpendicular to direction of travelling
 - Resultant force upwards so deceleration
 - Air resistance falls as speed falls
 - Decelerates until air resistance get as big as speed so the object falls at uniform speed again
 - Graph
 - Gradient should start with gradient 9.81 m s^{-2} not bigger than 9.81 m s^{-2}



- (Same to other situations moving through a fluid - resistance increase until the maximum speed is reached)
- Factors affecting terminal velocity
 - Higher mass → higher acceleration → higher terminal velocity
 - Higher volume / CSA → more air resistance → less acceleration → lower terminal velocity

3.4.1.5 Newton's laws of motion

- Newton's 1st law of motion
 - If no resultant external force are acting on a body, it will
 - If at rest, remain at rest
 - If moving, keep moving at constant speed in a straight line
- Newton's 2nd law of motion
 - The acceleration of an object is proportional to the resultant force experienced by the object
 - **Acceleration is in the same direction as the resultant force**
 - resultant force = mass × acceleration
 - $F = ma$
- Newton's 3rd law of motion

- When two objects interact, they exert **equal and opposite** forces on each other

3.4.1.6 Momentum

- Momentum calculation
 - Momentum = mass \times velocity
 - $p = mv$
- The principle of conservation of momentum
 - Momentum is always conserved for a system of interacting objects provided that no external resultant force acts on the system
 - Total final momentum = total initial momentum
- Types of collisions
 - Elastic
 - There is no loss of kinetic energy during the collision
 - Both momentum and KE are conserved
 - $m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$
 - Inelastic: only momentum is conserved, some KE is lost
 - Stick together: $m_1u_1 + m_2u_2 = (m_1 + m_2)v_{1+2}$
 - Colliding objects have less KE after the collision than before the collision
- Explosion
 - $m_1v_1 + m_2v_2 = 0$
 - KE of the objects has increased
- Newton's 2nd law of motion using momentum
 - The rate of change of momentum of an object is equal to the resultant force on it
 - $F = \frac{\Delta(mv)}{\Delta t}$
- Impulse
 - The change in momentum
 - Impulse = $F\Delta t = \Delta(mv)$
- Force-time graph
 - Area = $F\Delta t =$ change in momentum
- Stopping distances
 - Thinking distance $s_1 =$ speed \times reaction time = ut_0
 - Braking distance $s_2 = \frac{u^2}{2a}$
 - Stopping distance = $s_1 + s_2 = ut_0 + \frac{u^2}{2a}$
- Contact and impact time
 - impact time = $\frac{2s}{u + v} = \frac{2 \times \text{distanced moved by cars}}{\text{initial velocity} + \text{final velocity (of the same car)}}$
 - $a = \frac{v - u}{t}$
 - $F = ma = \frac{mv - mu}{t}$
 - (These calculations only need to be applied onto one car)
- Why airbags / seatbelts / etc. work
 - With no seat belt / airbag / etc. the person would not start to change their momentum until they hit the dashboard or windscreen
 - The person comes to stop quickly (short impact time)
 - Large change of momentum in a short time = large resultant force = large injury ($F = \frac{\Delta(mv)}{t}$)
 - With the seatbelt / airbag / etc. they will have a longer impact time (comes to stop more slowly)
 - They will experience a smaller resultant force and so less injury

3.4.1.7 Work, energy and power

- Work

- Work done = force \times distance moved in the direction of the force
- Unit = joules (J)
- $W = Fs \cos \theta$ = force \times displacement \times angle between force and direction of motion
- Force-displacement graphs
 - Area under line = work done
- Power
 - Rate of doing work = rate of energy transfer
 - $P = \frac{\Delta E}{\Delta t} = \frac{\Delta W}{\Delta t} = Fv \cos \theta$ = driving force \times velocity \times $\cos \theta$
- Efficiency
 - Efficiency = $\frac{\text{Useful work done}}{\text{Total energy input}} = \frac{\text{Useful energy output}}{\text{Total energy input}} = \frac{\text{Useful power output}}{\text{Total power input}}$
 - Can be expressed as a percentage

3.4.1.8 Conservation of energy

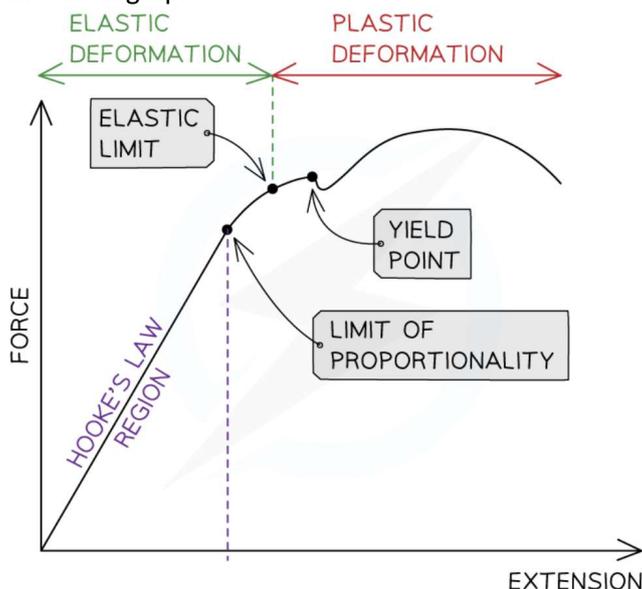
- Principle of conservation of energy
 - Energy cannot be created or destroyed but transferred from one store to another
- Kinetic energy
 - $E_k = \frac{1}{2}mv^2$
- (Gravitational) potential energy
 - $E_p = mg\Delta h$

3.4.2 Materials

3.4.2.1 Bulk properties of solids

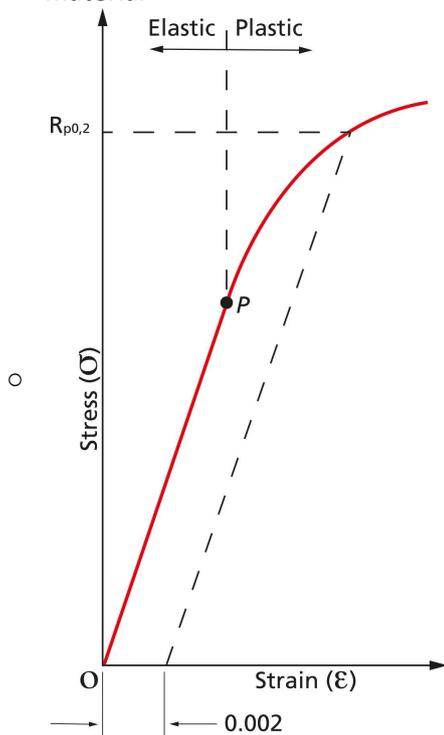
- Density
 - Mass per unit volume
 - $\rho = \frac{m}{V}$
- Hooke's law
 - The extension of the material is directly proportional to the load applied up to the limit of proportionality
 - $F = k\Delta L = \text{spring constant (stiffness)} \times \text{extension}$
- ★ • Springs combined together
 - Springs in parallel
 - $k = k_1 + k_2 + \dots + k_n$
 - Springs in series
 - $\frac{1}{k} = \frac{1}{k_1} + \frac{1}{k_2} + \dots + \frac{1}{k_n}$
- Energy transfer in springs
 - Hung vertically and stretched: KE / GPE → elastic strain energy
 - Force removed: elastic strain energy → KE → GPE

- Force-extension graph

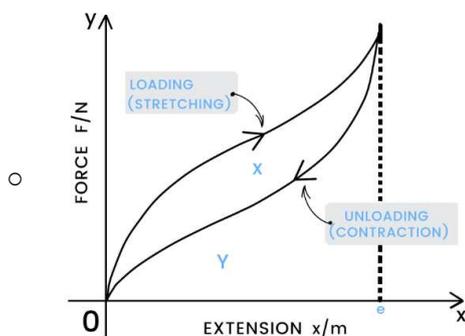


- Limit of proportionality: the point beyond which Hooke's Law is no longer true
- Elastic limit: the point beyond which the material will be permanently deformed, right after limit of proportionality
- Brittle materials: extend very little before it breaks / fractures at a low extension
- Plastic materials: experience a large amount of extension as the load is increased, especially after the elastic limit
- Types of stretches
 - Elastic
 - Material **returns to original shape** once force is removed
 - All the work done is stored as elastic strain energy
 - Plastic
 - Material **does not return to original shape** once force is removed
 - Work is done to move atoms apart so some energy is not stored as elastic strain energy but dissipated as heat
- Types of deformation
 - Tensile deformation
 - Deformation that **stretches** an object

- Compressive deformation
 - Deformation that **compresses** an object
- Strain energy
 - The area under the force-extension graph = work done to stretch the energy = strain energy
- Elastic strain energy in springs
 - $E_p = \frac{1}{2}F\Delta L = \frac{1}{2}k(\Delta L)^2 = \text{area under force-extension graph}$
- Loading & unloading curve for different materials
 - **Area between loading and unloading curves = work done to permanently deform the material or dissipated as heat**
- Plastic deformation
 - When a material is stretched beyond its elastic limit it will not return to its original length after the load is removed (permanent extension / deformation)
- Metal wire
 - Loading (the proportional part) + unloading curves have the same gradient
 - Gradient of the unloading line remains the same because the stiffness only changes with material

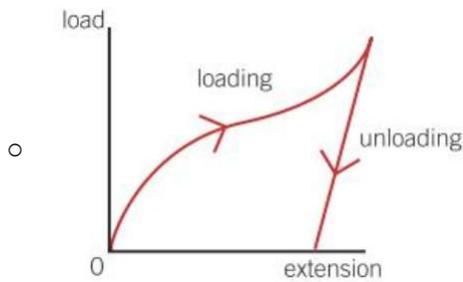


- Extension of elastic materials graph e.g. seatbelts
 - Loading and unloading curves are not linear & not the same
 - Returns to the original length when unloaded

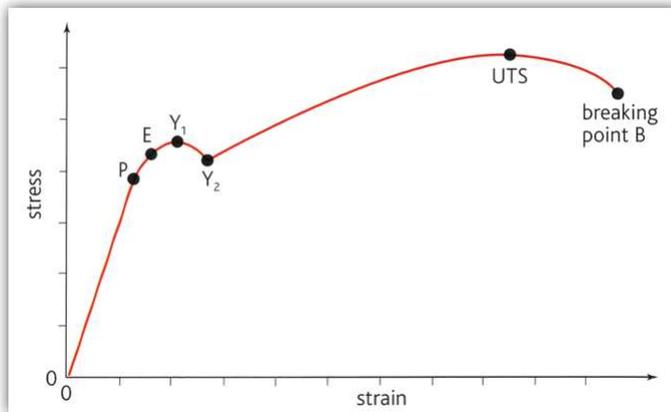


- Extension of plastic materials graph
 - The loading curve is not linear
 - During unloading the change in length is greater for a given change in tension
 - Does not return to its original strength

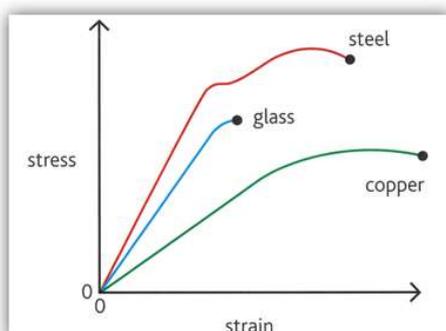
load | loading 1



- Stress-strain curves



- Before limit of proportionality (P)
 - **Gradient = Young modulus of the material**
 - Tensile stress \propto tensile strain
 - The material obeys Hooke's law
 - Elastic Limit (E)
 - Up to this point the material returns to its original length when the load acting on it is completely removed
 - Beyond this limit the material doesn't return to its original position after unloading and a plastic deformation starts to appear in it
 - Yield point (Y_1 and Y_2)
 - The stress at which the material starts to deform plastically
 - After the yield point is passed, **plastic deformation occurs**
 - 2 yield points: upper (Y_1) + lower yield point (Y_2)
 - At the upper yield point the wire weakens temporarily
 - At the lower yield point a small increase in stress causes a large increase in strain and the wire undergoes plastic flow
 - Ultimate tensile stress (UTS)
 - The maximum stress a material can withstand
 - After UTS the wire loses its strength, extends and **becomes narrower at its weakest point**
 - **Plastic deformation stops**
 - Fracture / breaking point (B)
 - The point in the stress-strain curve at which the material breaks / fractures
- Stress / strain curve for different materials



- Brittle materials (e.g. glass) snap without noticeable yield

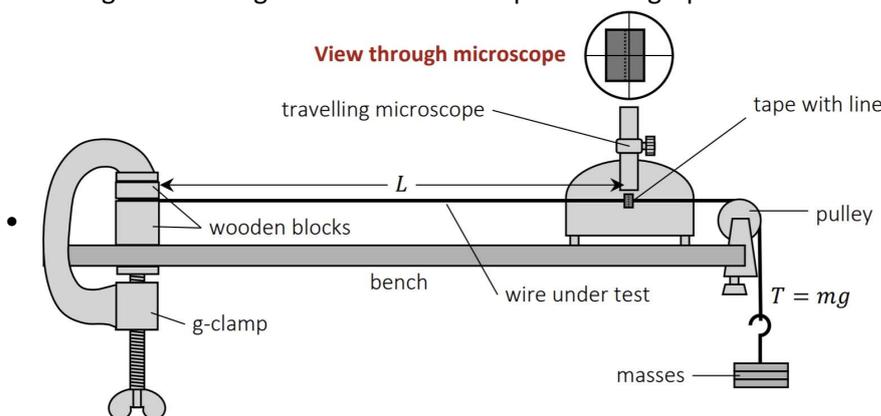
- Ductile materials can be drawn into a wire (e.g. copper)

3.4.2.2 The Young modulus

- Tensile strain
 - Extension per unit length
 - $\sigma = \frac{F}{A}$
- Tensile stress
 - The force per unit cross-sectional area
 - $\varepsilon = \frac{\Delta L}{L}$
- Young modulus
 - $E = \frac{\text{tensile stress}}{\text{tensile strain}} = \frac{FL}{A\Delta L}$
 - Unit = Pascal (Pa)
 - Measures the **stiffness of the material** (higher Young modulus = stiffer material)
 - * Young modulus is specific to the material and **doesn't change**
 - * Young modulus = gradient of the straight line part of the stress strain graph

Required practical 4 - determining the Young modulus

- Method
 - Measure the initial length of wire with a ruler
 - Measure the initial diameter of wire with a micrometer
 - Measure in several places and take mean
 - Diameter is very small so it cannot be measured by a ruler but only with a micrometer
 - Mark a cross onto the wire with a tape
 - Align the travelling microscope with the cross
 - Extension can be very small so a microscope is needed
 - Add load and align the travelling microscope with the cross again
 - Read off the extension of the wire
 - Repeat for a range of loads
 - Repeat up to the limit of proportionality / elastic limit
 - Repeat the experiment 2 more times for each value of load and calculate mean extension
 - Calculate tensile stress and strain for each load value
 - Plot a graph of stress against strain
 - Young modulus = gradient of the linear part of the graph



- Measurements
 - Length of the wire between clamp and mark (metre rule)
 - Diameter of the wire (micrometer, measure several positions + mean taken)
 - Extension of wire for a known mass (by moving travelling microscope and checking the scale)
 - Repeat readings for increasing load

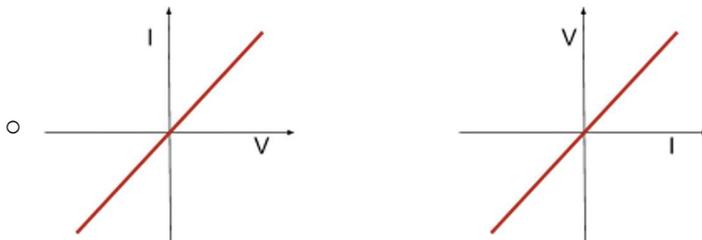
3.5.1 Current electricity

3.5.1.1 Basics of electricity

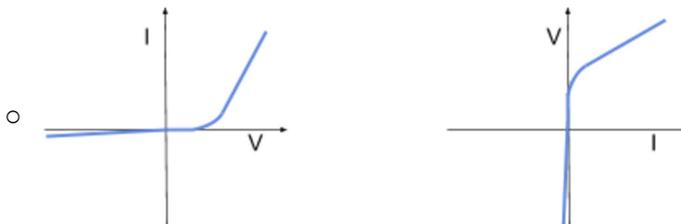
- Charge
 - Measured in coulomb (C)
 - ★• Charge of 1 electron = -1.60×10^{-19} C
 - Scalar quantity
- Electric current (I)
 - The rate of flow of charge
 - $I = \frac{\Delta Q}{\Delta t}$
- Potential difference (V)
 - The energy transferred per unit charge between two points in a circuit
 - When a charge of 1 C passes through a p.d. of 1 V, it does 1 J of work
 - $V = \frac{W}{Q}$
- Resistance (R)
 - A measure of how difficult it is for charge carriers to pass through a component
 - $R = \frac{V}{I}$
- Capacity
 - A measure of the total amount of charge which the battery can push around a circuit
 - Commonly measured in ampere-hours (A h)
 - ★• 1 Ah = a current of 1 A can flow for 1 hour = 3600 C
- Types of charge carriers
 - Insulator
 - Each electron is attached to an atom and cannot move away from the atom
 - Metallic conductor
 - Most electrons are attached to metal ions but some are delocalised
 - Delocalised electrons can carry charge through the metal
 - When a voltage is applied across the metal these conduction electrons are attracted towards the positive terminal of the metal
 - Semiconductor
 - Number of charge carriers increase with an increase of temperature (electrons break free from the atoms of the semiconductor)
 - Resistance fall as temperature rise

3.5.1.2 Current-voltage characteristics

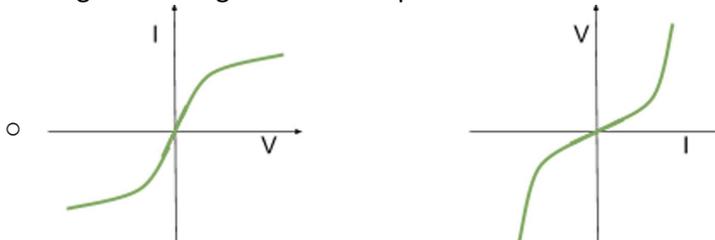
- Ohm's law
 - The current through a conductor is directly proportional to the potential difference across the conductor provided that temperature and other physical conditions remain constant
 - $V = IR$
 - * **Not** the definition of voltage
- Types of different conductors
 - Ohmic conductor
 - Follows Ohm' s law
 - Constant resistance as long as temperature and other physical conditions remain constant
 - Current-voltage graph will look like a straight line through the origin



- Semiconductor diode
 - Only lets current flow in one direction, converts AC to DC
 - Forward biased: allow current to flow easily past the threshold voltage (smallest voltage needed to allow current to flow)
 - Reverse biased: the resistance of the diode is extremely high so that only a very small current can flow



- Non-ohmic conductors e.g. filament lamp
 - Ohm's law obeyed initially (a straight line initially)
 - Does not have a constant resistance
 - As voltage increases current increases
 - More electrons flow through the wire per second
 - Current heats filament
 - Higher **rate** of collisions between ions in the lattice structure and electrons
 - Conducting electrons slow down more and lose more kinetic energy so current falls and resistance increases
 - Rate of increase of current is less than if resistance was constant
 - As current or voltage increases resistance increases so the gradient is not constant
 - Negative voltage and current produces the same effect

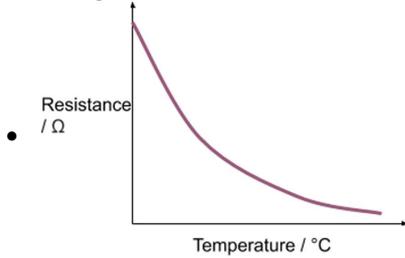


- ★ • Assumptions
 - Assume ammeters and voltmeters are ideal unless otherwise stated
 - Ammeters can be assumed to have zero resistance
 - Voltmeters can be assumed to have infinite resistance

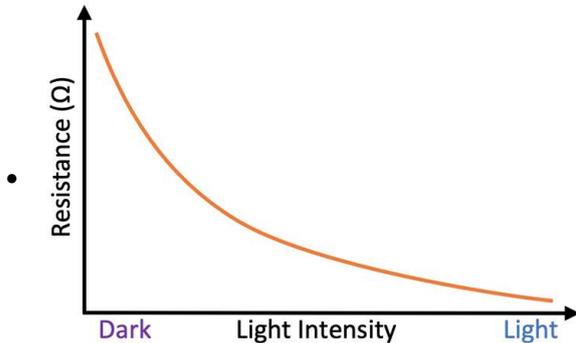
3.5.1.3 Resistivity

- Resistivity (ρ)
 - Resistance per unit length \times area of cross section
 - $\rho = \frac{RA}{L}$ or $R = \frac{\rho L}{A}$
 - Unit = $\Omega \text{ m}$
- Effect of temperature on the resistance of metal conductors
 - When the temperature of a metal conductor increases its resistance will increase
 - Metal ions gain KE from heating and vibrate more so they take up more space
 - More collisions between electrons and metal ions **per second** so they slow down more
 - Current falls so resistance increases
- Effect of temperature on the resistance of thermistors
 - When the temperature of a thermistor increases, its resistance will decrease
 - Increasing the temperature of a thermistor causes electrons to be emitted from atoms = more

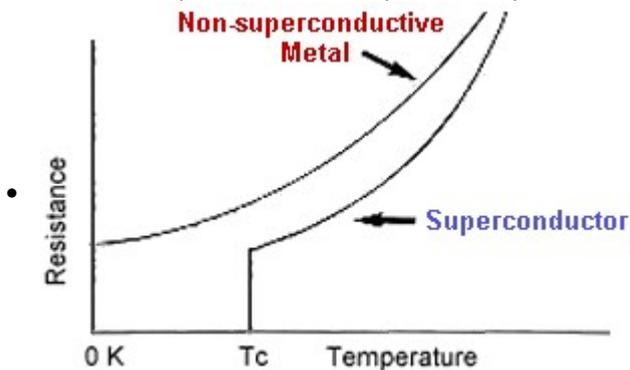
charge carriers = current increase



- Application of thermistors
 - Temperature sensors
 - Trigger an event to occur once the temperature drops below or reaches a certain value
 - e.g. turn on the heating once room temperature drops below a specific value
- LDR
 - Resistance decreases as light intensity increases
 - Used to trigger certain events



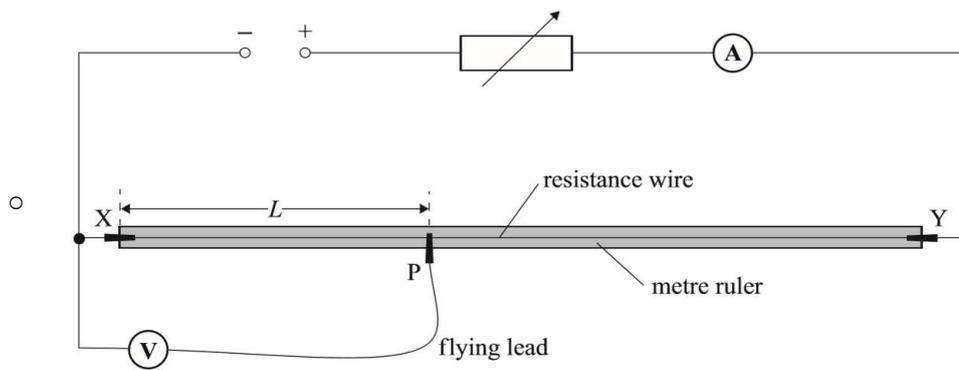
- Superconductivity
 - A property of certain materials which have **zero resistivity** at and below a critical temperature (T_c) which depends on the material
 - Resistivity decreases as temperature decreases
 - Zero resistivity = zero resistance
 - Critical temperature normally extremely low (close to 0 K)



- Applications of superconductors
 - Power cables
 - Reduce energy loss due to heating to zero during transmission
 - Production of strong magnetic fields
 - Do not require a constant power source
 - Used in maglev trains / certain medical applications
- Resistance of a wire
 - Normally assumed to be 0 so no PD is lost between 2 points on a wire with no resistors between them
 - The assumption can break down if the current is high / resistance in the rest of the circuit is low

Required practical 5 - determining wire resistivity

- Method
 - Set up the circuit as shown



- Connect the flying lead to the wire so that 0.10 m of the wire has its resistance measured
- Switch on the power supply and adjust the voltage of it so that the current in the circuit is 0.50 A
- Turn off the power supply between readings so the wire does not heat up and increase in resistance
- Measure and record the length and voltage across the wire by taking reading on the voltmeter
- Move the flying lead to increase the length by 0.10 m and repeat the measuring process for lengths up to 1.00 m
- Repeat the experiment twice for each reading and calculate an average voltage at each length
- Calculate resistance at each length by $R = \frac{V}{I}$
- Plot a graph of resistance against length
- Resistivity = gradient \times cross sectional area (gradient = $\frac{\rho}{A}$)
- Errors
 - Random Errors
 - The current flowing through the wire will cause its temperature to increase and increase its resistance and resistivity
 - Only allow small currents to flow through the wire so the temperature is kept constant and low
 - The power supply should be switched off between readings so its temperature doesn't change its resistance
 - Make at least 5-10 measurements of the diameter of the wire with the micrometer screw gauge and calculate an average diameter to reduce random errors in the reading
 - The wire should be free from kinks and held straight so the measurement of the length is as accurate as possible.
 - Systematic errors
 - Zero error when measuring wire length
- Safety Considerations
 - When there is a high current, and a thin wire, the wire will become very hot
 - Make sure never to touch the wire directly when the circuit is switched on
 - Switch off the power supply right away if you smell burning
 - Make sure there are no liquids close to the equipment, as this could damage the electrical equipment

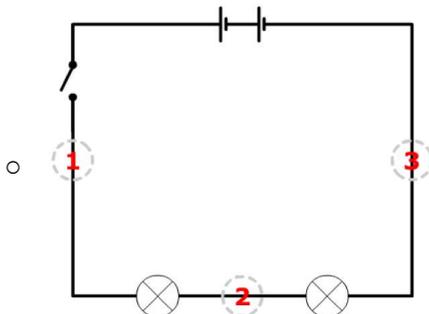
3.5.1.4 Circuits

- Circuit symbols

wires joining	wires crossing	lamp	ammeter	voltmeter
switch	cell	battery (several cells)	DC power supply	AC power supply
resistor	variable resistors	thermistor	thermistor	light-dependent resistor
heater	fuse	transformer	diode	light-emitting diode
earth	motor	generator	relay coil and switch	bell

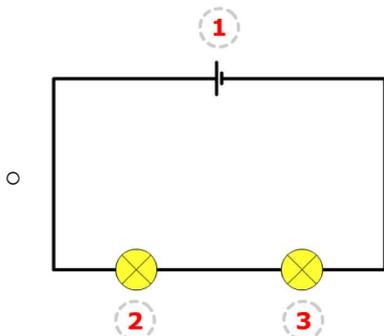
- Series circuit properties

- The current is the same at all points



- $\textcircled{1} = \textcircled{2} = \textcircled{3}$

- The sum of potential differences across the components is equal to the total EMF of the power supply

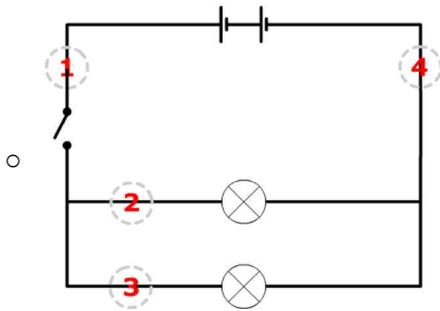


- $\textcircled{1} = \textcircled{2} + \textcircled{3}$

- Parallel circuit properties

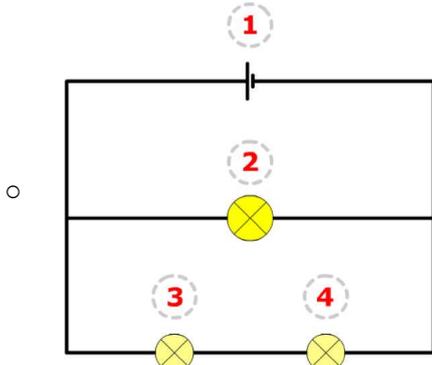
- The current splits up

- Some of it going one way and the rest going the other
 - Total current in the circuit = sum of the currents in the branches



○ $\textcircled{1} = \textcircled{2} + \textcircled{3} = \textcircled{4}$

- Total voltage of a parallel circuit has the same value as the voltage across each branch

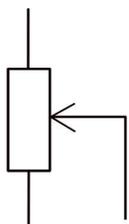


○ $\textcircled{1} = \textcircled{2} = \textcircled{3} + \textcircled{4}$

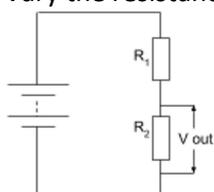
- Total voltage of cells
 - Cells joined in series
 - $V_T = V_1 + V_2 + V_3 + \dots$
 - Identical cells joined in parallel
 - Total voltage = voltage of one cell as current is split equally between branches so overall pd is the same as if the total current was flowing through a single cell
 - $V_T = V_1 = V_2 = V_3 = \dots = \varepsilon - \frac{Ir}{n}$
 = emf - $\frac{\text{total current of the circuit} \times \text{internal resistance of each cell}}{\text{number of cells}}$
 - Total internal resistance = calculated in the same way as other parallel circuits
 - Act like one cell but with reduced internal resistance
- Advantages of cells joined in parallel
 - Reduce the combined internal resistance of the cells - reduce lost volts
 - Less power drawn from each cell so it lasts longer
- Total resistance calculation
 - In series
 - $R_T = R_1 + R_2 + R_3 + \dots + R_n$
 - In parallel
 - $\frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \dots + \frac{1}{R_n}$
- Power (P) and energy (E)
 - $P = IV = \frac{V^2}{R} = I^2R$
 - $E = Pt = IVt$
- Kirchhoff's laws
 - In **DC** circuits
 - Kirchhoff's first law (conservation of charge)
 - The total current flowing into a junction is equal to the current flowing out of that junction
 - No charge is lost at any point in the circuit
 - Kirchhoff's second law (conservation of energy)
 - The sum of all the voltages in a series circuit is equal to the battery voltage
 - No energy is lost at any point in a circuit

3.5.1.5 Potential divider

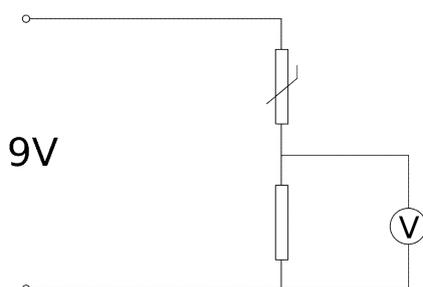
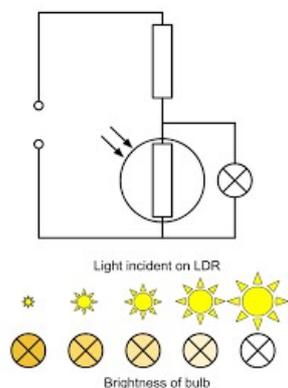
- Potential divider
 - Circuits which produce an output voltage as **a fraction of its input voltage**
 - Has several resistors in series connected across a voltage source
 - Used to supply constant or variable potential difference from a power supply



- Using variable resistors
 - Potential divider supply a variable pd
 - Use variable resistor as one of the resistor in series
 - Vary the resistance across = vary pd output



- Using thermistor / LDR
 - Resistance decreases as temperature / light intensity increases
 - Used to trigger certain events



- Change in voltage across one resistor
 - Resistance stayed constant
 - Current increase / decrease → voltage across changes
 - Resistance changed
 - The resistor has increased / decreased share of total resistance
 - New current is the same in both resistors
 - The resistor gets a larger / smaller share of the EMF

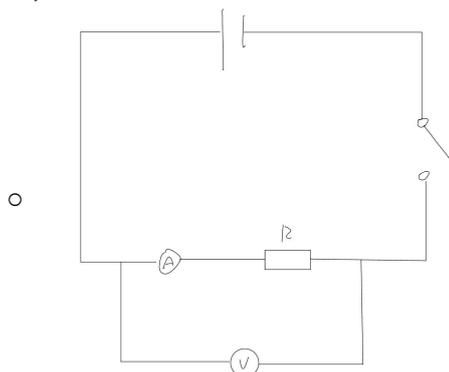
3.5.1.6 Electromotive force and internal resistance

- Internal resistance of batteries
 - The resistance of the materials within the battery
 - Caused by electrons colliding with atoms inside the battery so some energy is lost before electrons leave the battery
 - Represented as a small resistor inside the battery
- Terminal pd (V)
 - Pd across the resistor(s)
 - $V = \epsilon - Ir$
- Lost volts (v)
 - Pd across the internal resistor in the battery

- = energy wasted by the cell per coulomb of charge
- Electromotive force
 - The energy converted (from chemical) to electrical energy by a cell for per coulomb of charge that passes through it
 - Can be measured by measuring the voltage across a cell using a voltmeter when there is no current running through the cell
 - $\varepsilon = \frac{E}{Q} = \frac{\text{electrical energy transferred}}{\text{charge}}$
 - $\varepsilon = V + v = I(R + r) = \text{current} \times (\text{load resistance} + \text{internal resistance})$

Required practical 6 - finding the EMF and internal resistance of a cell

- Method
 - Set up the circuit as shown above with 2 (1.5V) cells connected in series

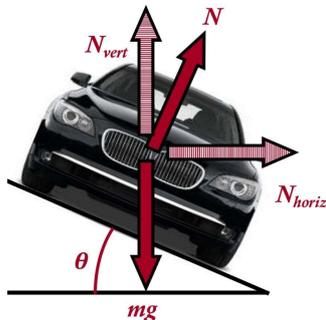


- Connect a voltmeter across the resistor to measure the load voltage
- Close the switch so that the current flows in the circuit
- Record the ammeter and voltmeter readings
- Open the switch to cut off the current and prevent heating in the circuit
- Replace the resistor with a different resistor with a different resistance and repeat the measuring process
- Use at least 5 different resistors with different resistances
- Repeat the experiment 2 more times for each resistor and calculate the mean current and voltage
- Plot a graph of load voltage against current
- EMF of the cell is the y-intercept of the graph while the internal resistance of the cell is the magnitude of the gradient of the graph
- Safety
 - Another resistor can be included in series with the other to avoid high currents which could be dangerous and make the wires get hot
- Improvements / controls
 - Only close the switch for as long as it takes to read off each pair of readings
 - Prevent the internal resistance of the battery or cell from changing during the experiment due to heating
 - Use fairly new batteries/cells
 - The emf and internal resistance of run down batteries can vary during the experiment

3.6.1 Periodic motion

3.6.1.1 Circular motion

- Why does centripetal force exist
 - An object in a circular motion is constantly changing direction
 - Its velocity is changing hence it is accelerating
 - Acceleration requires a resultant force = centripetal force
- Angular speed / velocity
 - $\omega = \frac{d\theta}{dt} = \frac{2\pi}{T} = 2\pi f = \frac{v}{r}$
 - Unit = rad s^{-1}
 - All points in a rotating object have the **same angular velocity**
- Tangential / linear speed
 - $v = r\omega$
 - Unit = m s^{-1}
- Centripetal acceleration
 - $a = \frac{v^2}{r} = \omega^2 r$
- Centripetal force
 - The force that acts towards the centre of the circular path i.e. acts perpendicular to the direction of motion
 - $F = ma = \frac{mv^2}{r} = m\omega^2 r$
- Banked tracks
 - Provide extra centripetal force via normal reaction force (horizontal component is towards the centre)
 - In horizontal track friction is the only source of centripetal force

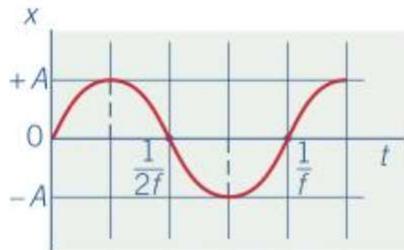


- Roller coaster
 - Head pointing downwards: some centripetal force provided by the weight = less normal force
 - Head pointing upwards: more support force needed to counter the weight and provide centripetal force

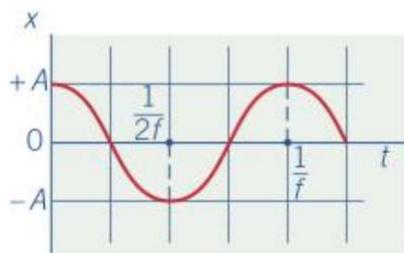
3.6.1.2 Simple harmonic motion (SHM)

- Simple harmonic motion
 - The acceleration of the object is proportional to the displacement in the **opposite direction**
 - Acceleration is directed towards the mean position
 - Defining equation for SHM: $a = -\omega^2 x$
- Period
 - Time for one **complete** oscillation
- Amplitude
 - Maximum displacement from equilibrium position
- Mathematical solutions
 - Starting at 0 displacement
 - $x = A \sin \omega t$

- $v = A\omega \cos \omega t$
- $a = -A\omega^2 \sin \omega t$
- Starting at maximum displacement
 - $x = A \cos \omega t$
 - $v = -A\omega \sin \omega t$
 - $a = -A\omega^2 \cos \omega t$
- Velocity
 - $v = \pm\omega\sqrt{A^2 - x^2}$
- Maximum speed and acceleration
 - Maximum speed = ωA
 - Maximum acceleration = $\omega^2 A$

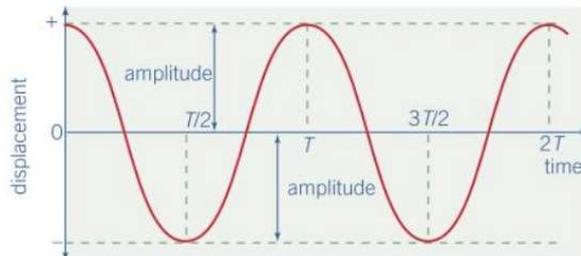


$$x = A \sin(\omega t)$$

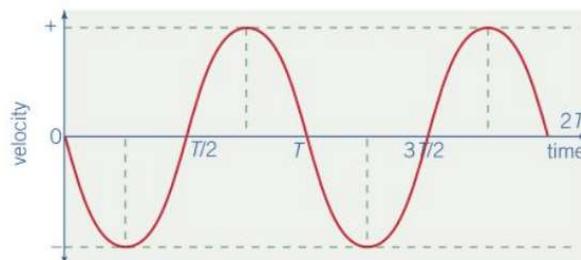


$$x = A \cos(\omega t)$$

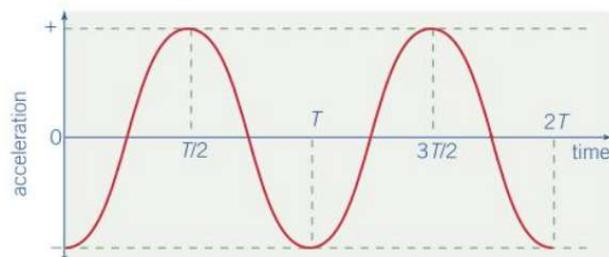
- Motion graphs for SHM



▲ Figure 2(i) Displacement against time



▲ Figure 2(ii) Velocity against time



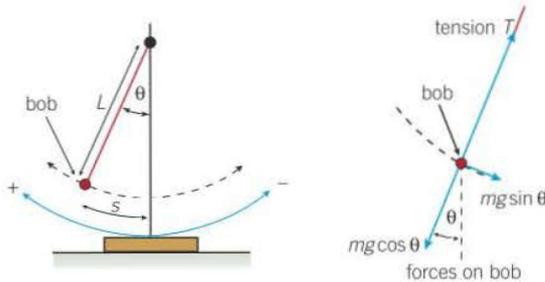
▲ Figure 2(iii) Acceleration against time

- Gradient of d-t graph = v-t graph

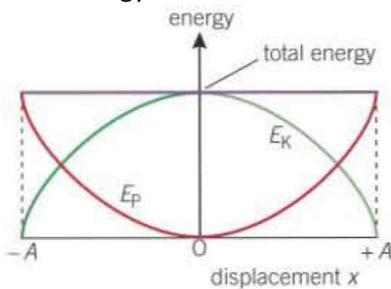
- Gradient of v-t graph = a-t graph
- Shifts to the left by $\frac{\pi}{2}$ each time

3.6.1.3 Simple harmonic systems

- Free oscillation
 - No energy lost during oscillations



- Energy changes
 - Energy $\propto A^2$
 - $E = KE + PE$
 - Kinetic energy
 - Minimum when $x = \pm A$
 - Maximum when $x = 0$
 - Potential energy
 - Minimum when $x = 0$
 - Maximum when $x = \pm A$
 - Total energy remains constant unless being forced / damped



- Mass-spring system

- $\omega = \sqrt{\frac{k}{m}}$
- $T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{m}{k}}$
- $E_k = \frac{1}{2}kA^2 - \frac{1}{2}kx^2$

- Simple pendulum

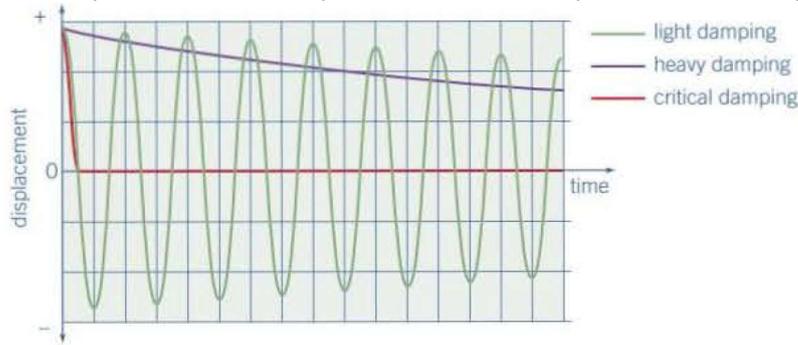
- $\omega = \sqrt{\frac{g}{l}}$
- $T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{l}{g}}$

- Only works when θ does not exceed approx. 10° / amplitude is small

- Damped oscillations

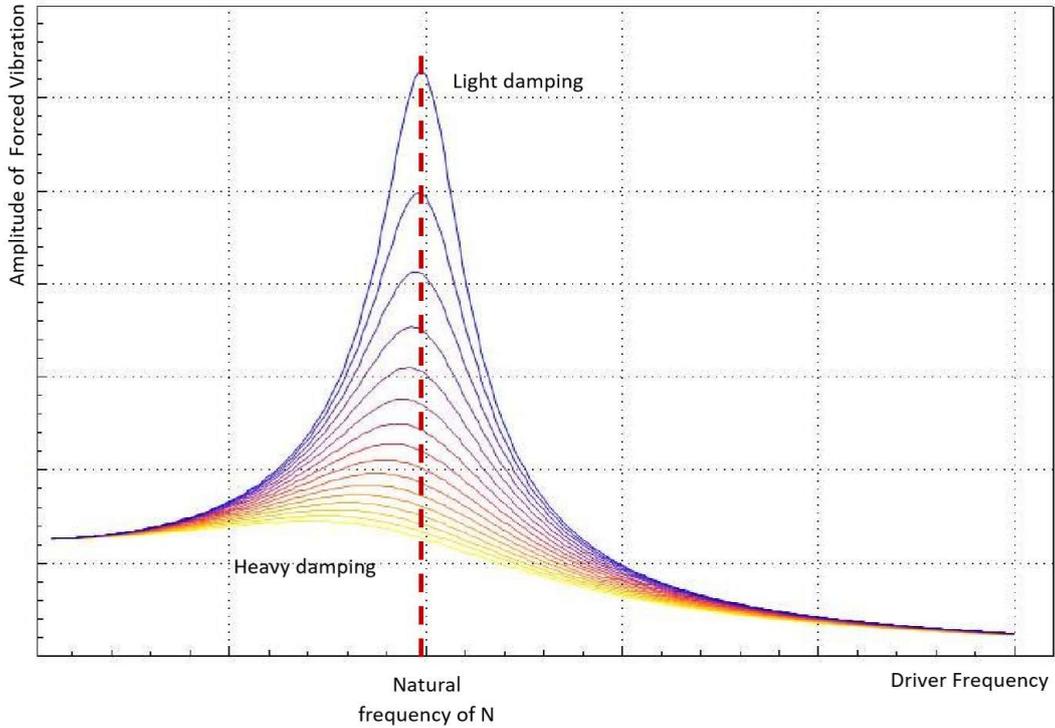
- Oscillations that reduce in amplitude due to the presence of resistive forces
- Lightly damped system
 - Amplitude decrease exponentially over time
 - Constant frequency
 - $A = A_0 e^{-Ct}$ (C is an arbitrary constant, depends on level of damping)
- Critically damped system
 - System returns to equilibrium in the **shortest** possible time without oscillation
- Heavily damped system

- System returns to equilibrium more slowly than critical damping without oscillating



3.6.1.4 Forced vibrations and resonance

- Free vibrations
 - No damping → the amplitude of the oscillations is constant
 - No periodic force acting on the system other than internal forces
 - No energy input
- Forced vibrations
 - System made to oscillate by a periodic external force / energy source / another oscillator it is connected to
 - Oscillator is acted on by a periodic external force
 - Energy is given periodically by an external source
 - Made to oscillate at the frequency of another oscillator
- Natural frequency (f_0)
 - The frequency that an object oscillates at if it is displaced from its equilibrium position
- Resonance
 - Applied frequency of the periodic force = the natural frequency of the system (state what is applied frequency / what produces it)
 - Phase difference between system and the periodic force = $\frac{1}{2}\pi$
 - Energy is transferred to the system more efficiently so the system gains max KE
 - The driving force continually supplies energy to the system
 - The amplitude of the oscillator increases
 - The amplitude would increase indefinitely if no resistive forces are present
- Resonant frequency
 - The frequency with the maximum oscillating amplitude (the oscillating system in resonance)
- Effect of damping on resonance frequency
 - Damping reduces the resonance frequency
 - When a system is damped, a resistive force is acting against the restoring force so the object travels slower → lower resonance frequency
- Amplitude-driving frequency graphs



- - More damping = smaller maximum amplitude at resonance
 - Less damping = sharpness of the peak amplitude of the curve increases, larger maximum amplitude
 - The closer the driving frequency is to the natural frequency the larger the amplitude
- Barton's pendulums
 - The driver pendulum is displaced and released so that it oscillates in a plane perpendicular to the plane of the pendulums at rest
 - All pendulums swing at a very small amplitude except the one which has the same / similar length to the driver pendulum
 - $T = 2\pi \sqrt{\frac{l}{g}}$
 - Only that pendulum's frequency matches the natural frequency of the driver pendulum
 - $\frac{\pi}{2}$ out of phase with the driver pendulum
 - Oscillates in resonance with the driver pendulum

